Numerical bifurcation using time steppers

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Some of the more interesting features of nonlinear dynamical systems of the form

$$\frac{d\mathbf{u}}{dt} = \mathbf{f}(\mathbf{u}; \lambda) \tag{1}$$

are equilibrium solutions and their stability. Numerical bifurcation techniques construct special extended systems or bordered systems to compute solutions of

$$\mathbf{f}(\mathbf{u}; \lambda) = 0 \tag{2}$$

and to determine how the number and stability of equilibrium solutions changes as the parameter λ is varied. Frequently, especially in areas of applied science, considerable time and effort has been invested in developing numerical codes to solve (1) efficiently, and there is little enthusiasm for repeating this effort to solve (2). This necessarily limits the solutions that can be determined numerically to those that are stable, making a complete understanding of the bifurcation structure difficult, and behavior near bifurcation points very expensive to obtain. The Recursive Projection Method and Newton-Picard iteration are designed to overcome these problems. I will illustrate each of these two related methods using a simple buckling problem whose bifurcation structure is completely understood.