Mathematics 667 **Homework (due Mar. 4)** A. Hulpke

17) In $R = K[x, y]$ let $I = (x^3, xy) \triangleleft R$.
a) Show that for any natural number n, the ideal (x^3, xy, y^n) is primary.
b) Show that $I = (x) \cap (x^3, y)$ is a minimal primary decomposition.
c) Construct infinitely many different minimal primary decompositions of I. What are associated and embedded primes?

18) Let $R = K[\underline{\mathbf{x}}]$ and $I \triangleleft R$ a primary ideal. Show that the radical of I is the unique smallest prime ideal containing I.

19) a) Show that if a prime ideal contains the intersection of finitely many ideals, it must contain one of intersectants.

20) Let $A = \mathbb{Z}$ and M a finitely generated abelian group (i.e. a $\mathbb{Z}$-module). Identify the associated primes of M in terms of the structure theory for finitely generated abelian groups.