59) (This came up in my research last month...)
For $n \geq 1$, $m > 1$ reduction modulo m gives a homomorphism $\varphi\colon \mathrm{SL}_n(\mathbb{Z}) \to \mathrm{SL}_n(\mathbb{Z}/m\mathbb{Z})$. ($\mathrm{SL}_n(\mathbb{Z}) = \{M \in \mathbb{Z}^{n\times n} \mid \det(M) = 1\}$.) However for a matrix $A \in \mathrm{SL}_n(\mathbb{Z}/m\mathbb{Z})$ the obvious preimage $\in \mathbb{Z}^{n\times n}$ does not neccessarily have determinant 1. (An example is given in part b))
a) Let $B \in \mathbb{Z}^{n\times n}$ such that $B \mod m = A$. Let $B = PDQ$ be the Smith normal form of B. What can you tell about $\det(B)$ and about Q? Show how to find a matrix $C \in \mathrm{SL}_n(\mathbb{Z})$ such that $C \mod m = A$.
b) Find such a matrix C for $m = 7$ and

$$A = \begin{pmatrix} 5 & 5 & 1 & 4 \\ 6 & 2 & 2 & 5 \\ 1 & 6 & 0 & 3 \\ 5 & 5 & 4 & 0 \end{pmatrix}.$$

60) Show that two 3×3 matrices are similar if and only if they have the same characteristic polynomial and the same minimal polynomial. Give a counterexample to this assertion for 4×4 matrices.

61) Determine the characteristic and the minimal polynomial of the following matrix over $\mathbb{F}_2$:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

62) Let

$$A := \begin{pmatrix} -3 & -5 & 6 \\ -16 & -19 & 24 \\ -16 & -20 & 25 \end{pmatrix}, \qquad B := \begin{pmatrix} 1 & 1 & -1 \\ 0 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

a) Determine the rational normal form of A and find an invertible matrix $Q \in \mathbb{Q}^{3\times 3}$ such that $Q^{-1}AQ$ is in rational normal form.
b) Determine the characteristic polynomial and the minimal polynomial of A.
c) Show that A and B are similar.
d) Find an invertible matrix $P \in \mathbb{Q}^{3\times 3}$ such that $P^{-1}AP = B$.

63) a) Let F be a field and let $A \in F^{n\times n}$. Show that $A \sim A^T$.
b) Show that there cannot be a single matrix $B \in \mathrm{GL}_n(F)$ such that $A^B = A^T$. (That is, the statement in a) is $\forall A \exists B : A^B = A^T$ and not $\exists B \forall A$.