45) Let $n$ be an integer and $m = n^2$. Show that an $m \times m$ Sudoku problem can only have a unique solution if all of the following conditions are fulfilled:

- It contains at least $m - 1$ numbers
- It contains numbers in $n(n - 1)$ different rows
- It contains numbers in $n(n - 1)$ different columns

(It has been proven in 2012 that there is no $9 \times 9$ Sudoku with less than 17 given numbers that has a unique solution.)

46) Find, using an implementation of the LLL algorithm, a combination of the numbers

$$276, 1768, 1993, 2536, 4251, 4884, 5020, 5347, 7401, 9072$$

that sums up to 33164.

47) Let $J = I^{n \times n}$ and $I$ the identity matrix. Show that

$$\det(xI + yJ) = (x + yn)x^{n-1}.$$  

**Hint:** Show that the eigenvalues of the matrix are $x$ (with multiplicity $n - 1$) and $x + ny$.

48) For $n > 2$ let $X$ be the points of $PG_n(q)$ and $B$ the $i$-flats (i.e. the $i + 1$ dimensional subspaces of $\mathbb{P}^{n+1}_q$). What are the parameters of such a design?

49) a) Why can’t there be a $4 - (8, 6, 1)$ design?

b) Suppose that $k - 1$ is a prime. Show that there are $2 - (v, k, \lambda)$ designs only if $k - 1 | v - 1$ or $k - 1 | \lambda$. 
