1) Let $G$ be the group of (rotational) symmetries of a regular tetrahedron. Determine its orbits on edges, and on pairs of edges. (The tetrahedron has 6 edges, thus there are $\binom{6}{2} = 15$ pairs).

2) Let $\Omega$ be the set of connected, labelled, graphs on 4 vertices. (For example $1 - 2 - 3 - 4$ is such a graph.) Determine the orbits of $S_4$ on $\Omega$, acting by renumbering vertices.

3) Let $\Omega = \{0, 1, 2, 3, 4\}$ and let $a$ act on $\Omega$ by addition of 1 modulo 5, $b$ act on $\Omega$ by multiplication with 2 modulo 5, and $G = \langle a, b \rangle$. (Both are affine transformations, thus they lie in a group.)

a) Write down permutations for $a$ and $b$ acting on $\Omega$.

b) Draw the Schreier graph for the action of $G$ on $\Omega$.

4) Let $G$ be a group and $H, K \leq G$. Show that the action of $G$ on the cosets of $H$ is equivalent (where we choose the homomorphism $G \to G$ to be the identity) to the action of $G$ on the cosets of $K$ (i.e. there is a bijection $\psi$ on the cosets such that $\psi((Hx)^g) = \psi((Hx)^s)$ if and only if $H$ and $K$ are conjugate in $G$, that is there exists $g \in G$ such that $K = H^g = g^{-1}Hg = \{g^{-1}hg \mid h \in H\}$).

5) The Figure below depicts an unfolded icosahedron with its faces labelled from 1 to 20.

a) Show that the group $G$ of rotational symmetries of the icosahedron has order 60.

b) Show that the permutations

$$p_1 = (1, 6, 8, 4, 10)(2, 5, 17, 13, 9)(3, 7, 15, 12, 19)(11, 20, 14, 18, 16)$$

$$p_2 = (1, 13, 19, 9, 6)(2, 4, 17, 18, 11)(3, 8, 10, 12, 16)(5, 15, 14, 20, 7)$$

represent rotational symmetries of this icosahedron along different axes.

c) Show that $p_1 \notin \langle p_2 \rangle$ and $p_2 \notin \langle p_1 \rangle$.

d) [For those who had abstract algebra] Show that any group of order $\frac{n}{60}$ must have a normal 5-Sylow subgroup. (Hint: In the case of order 30, show that if there were six 5-Sylow subgroups, the 3-Sylow subgroup must be normal. Then the subgroup generated by the 3-Sylow subgroup and a 5-Sylow subgroup must have order 15, thus be normal, and thus contain all six 5-Sylow subgroups, which is impossible, since 15 is not a multiple of 6.)

Conclude that $p_1$ and $p_2$ must generate the full group of all rotational symmetries.