Mathematics 366 **Homework (due Sep. 2)** A. Hulpke

1) If it is 3pm now, what time will it be in 1253 hours?

2) The easter formula of Gauß provides a way for calculating the date of easter (i.e. the first sunday after the first full moon of spring[1]) in any particular year. For the period 1900-2099 (a restriction due to the use of the Georgian calendar) a simplified version is:

- Suppose the year is Y.
- Calculate $a = Y \bmod 19$, $b = Y \bmod 4$, $c = Y \bmod 7$.
- Calculate $d = (19a + 24) mod 30$ and $e = (2b + 4c + 6d + 5) mod 7$
- If $d + e < 10$ then easter will fall on the $(d + e + 22)$th March, otherwise the $(d + e - 9)$th of April.
- Exception: If the date calculated is April 26, then easter will be April 19 instead. (There is a further exception for certain April 25 dates, again due to the Georgian calendar, which we ignore for simplicity.)

a) Calculate the easter day for 2010 and for 1968.
b) Show, by finding explicitly years for which this holds, that the date may lie on March 23. (March 22 is possible but does not occur between 1900 and 2099.)
Hint: First determine what values of d and e are necesarry for this. Then work backwards to find suitable values for a,b, c and determine the years. c) Assuming the formula would work for any year (not just 1900-2099), what is the smallest period, after which we can guarantee that the sequence of dates repeats? (This is called the *Paschal cycle*.)

3) a) Determine the check-digit for the ISBN 3-86073-427-?
b) Show (by giving an explicit example) that the check-digit of the ISBN does not necessarily correct multiple errors.

4) The method of casting out nines is used to verify addition or multiplication. In an calculation (e.g. $a + b = c$) we add up the digits in both a and b repeatedly, eliminating numbers that add up to 9, until we end up with a single digit. We do the same with c. If the results do not agree there must have been an arithmetic error. (For example if the operation is $1234 + 5678 = 6912$ we add for a and b: $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$, $3 + 6 = 9$, result 0. For c: $6 + 1 + 2 = 9$ result 0. Agreement.)
Explain (for the case of addition) why this method works.

5) Find by hand integers s and t such that $1 = 5s + 7t$. Show that s and t are not unique. What is (for a fixed pair (s, t)) the gcd of s and t?

6) Get accustomed with the system GAP.

[1] for western churches. Orthodox churches use a slightly different calendar that leads to different dates