Practice

§3.8: 3,7,14,20
§3.9: 2,3,10,18
§4.2: 13,17,22,24,30,35

Hand In

33) Show that the homogeneous solution of $my'' + \gamma y' + ky = 0$ approaches zero for any choice of the parameters m, γ, k, as long as they are all strictly positive.

34) Assume that the Earth is a solid sphere of uniform density, with mass M and radius R. For a particle of mass m within the Earth at distance r from the center of the Earth, the gravitational force attracting m towards the center is $F_r = -GM_r m/r^2$, where M_r is the mass of the part of the Earth within a sphere of radius r.
a) Show that $F_r = -GMmr/R^3$.
b) Now suppose that a small hole is drilled straight through the center of the Earth, connecting two antipodal points on its surface. Let a particle of mass m be dropped at $t = 0$ into this hole with initial speed zero. Let $r(t)$ denote its distance from the center of the Earth at time t. Conclude from Newton's second law that $r'' = -k^2 r$, where $k^2 = GM/R^3$.
c) Solve for $r(t)$.
d) Give a formula for the period of the motion.

35) Determine a general solution for the differential equation

$$\frac{\mathrm{d}^5 y}{\mathrm{d}t^5} - \frac{\mathrm{d}^3 y}{\mathrm{d}t^3} - \frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + y = 0$$

36*) Help Gandalf save Middle Earth from Sauron At Barad-dûr, Sauron wants to fire lava balls made from the lava of Mount Doom on Gondor, the only stronghold protecting Middle Earth from the Dark Lord. The cannon he uses is manned by the Nazgûl. If one of these balls hits Gondor, it will be destroyed and Middle Earth will be doomed forever.

a) Gandalf, having taken differential equations before, gets smart: he knows about natural frequencies and the Power of Resonance. He therefore starts hitting the ground in a periodic fashion such that

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + B^2 y = \cos(Gt),$$

where $y(t)$ is the oscillation of the ground, G is the Gandalf constant and B is the Barad-dûr constant. What does the Gandalf constant G have to be to cause resonance?

b) If G is chosen to cause resonance, solve for the ground oscillations $y(t)$, using that $y(0) = 0$ and $y'(0) = 0$.

c) To knock the Nazgûl Cannon off Barad-dûr , the amplitude of the oscillations needs to be greater than π (i.e., we need $y \geq \pi$). For $B = 1/2$, at what time t does Gandalf knock the cannon off?

d) Given that it takes the Nazgûl $t = 3.15$ to load the cannon, does Gandald save Middle Earth?

Problems marked with a *are bonus problems for extra credit.