Practice

§2.6: 1,2,16,18
§2.9: 1,3,5,7,9,10

Hand In

17) Consider the Differential equation

$$(y^2 + e^{x\cdot y}(1+(x\cdot y)) + (x^2 e^{x\cdot y} + 2x\cdot y)\frac{\mathrm{d}y}{\mathrm{d}x} = 0$$

a) Show that this differential equation is exact
b) Determine a general solution for this equation.

18) a) Consider a differential equation

$$M(x,y)\,\mathrm{d}x + N(x,y)\,\mathrm{d}y = 0 \tag{1}$$

Suppose that a function $\mu(x,y)$ is given such that it fulfills the partial differential equation

$$M(x,y)\cdot\partial_y\mu(x,y) + N(x,y)\cdot\partial_x\mu(x,y) + (\partial_y M(x,y) - \partial_x N(x,y))\cdot\mu(x,y) = 0$$

Show that equation(1) becomes exact when multiplied by μ. (We call μ an integrating factor.)
b) Consider the differential equation

$$(x+2)\sin(y)\,\mathrm{d}x + x\cos(y)\,\mathrm{d}y = 0$$

Show that $\mu(x,y) = xe^x$ is an integrating factor for this equation and determine a solution for $y(x)$.

19) A prospective homebuyer can afford to spend 1000$ every month for payment of a mortgage (interest and principal. Ignore taxes &c.) A bank offers him an interest rate of 6% for a 30-year mortgage.
a) What is the maximum mortgage he can afford to not exceed the monthly payment ?
b) To what level would the montly payment rise, if the interest rate rose to 7%?

20) Consider the initial value problem

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 5 - \sqrt{y+x}, \qquad y(0) = 10$$

a) Sketch a direction field (for example using Maple)
b) Determine numerically an approximation for $y(4)$, using a step width of 0.5. (i.e. $t_0 = 0$, $t_1 = 0.5$, $t_2 = 1$, $t_3 = 1.5$ &c.)