Practice

§2.3: 1,3,6,9,19,30
§2.5: 1,4,8,10,13,14

Hand In

12) A spherical pill and a cubical pill both have the same volume, and are dropped into a solute at the same time. Each dissolves at a rate proportional to the square root of their surface area. Which will completely dissolve first? Hint: you might want to find the radius as a function to time for the spherical pill, and sidelength as a function of time for the cubical pill. You should assume the proportionality constants are the same. Then find which reaches zero first.

13) At the time of the previous census (1995) the city of Fort Collins had a population of 80000 people. The last (2000) census revealed that the population of Fort Collins was 120000 people. The city planners do not wish to limit growth until the population reaches 180000. Assuming the rate of change of the population is proportional to the population, when will this occur?

14) A water tank is in shape of a paraboloid, generated by revolving the arc $y = x^2$ ($0 \leq x \leq 2$) about the x-axis (i.e. its maximum radius is $2m$ and its height is $2^2 = 4m$. The tank is filled to the brim with water and drains to a hole at the bottom.
According to Torricelli's Law, the volume draining fulfills the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -\rho\sqrt{y}$$

where $\rho = ab\sqrt{2g}$ where $a = 10^{-4}m^2$ is the area of the hole, $b = 0.6$ is a constant depending on the liquid and $g = 9.81\frac{m}{s^2}$ is the gravity constant.
a) As the volume depends on the height, rewrite the differential equation as an equation only involving the height $y(t)$ at a given time.
b) Solve this differential equation for the initial value $y(0) = 4m$.
c) How long will it take for the container to empty completely. (What unit will the result be in?)

15) (Maple)
Consider the differential equation

$$y' = ty(3-y)$$

a) Plot a direction field for this differential equation for $-1 < t < 6$ and $-5 < y(t) < 8$.
b) Use this direction field to sketch solutions for the initial value problems $y(1) = -1, 0, 1, 3.5, 4$.
For what values of t are these solutions valid?

16) An epidemic spreads through a population by contacts between sick and healthy individuals and the rate of spread is proportional (with a factor $0 \leq \alpha \leq \beta$) to the number of contacts. We also assume that individuals with a rate of $0 \leq \beta \leq \alpha$ become healthy again. The ratio of sick individuals thus fulfills the autonomous differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \alpha y(1-y) - \beta y$$

a) What are the equilibruum points of this differential equation. Which ones aree stable, semistable or instable?
b) What is the asymptiotic behaviour if $y(0) > 0$ and $t \to \infty$?

Computer use

Some of the homework problems are too hard to do by hand (and marked as such) and I recommend that you use the program Maple to solve them. Using the login I provided you have access to the computers in the Lab Weber 205 while it is not used.
If you want to use the program at home, for students in this course the student edition of Maple is available at a reduced price of $\$75$(+tax,shipping) under the URL (in a single line):

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Address not given on the web -- please ask me.
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(This URL is unique to the course and is restricted to the 35 students currently enrolled in this section.)