Mathematics 340-3 **Homework (due Sep. 9)** A. Hulpke

Note: Monday's office hour has to be moved to 3-4 due to a departmental commitment of mine. Apologies for the change!

Practice

§2.1: 13,16,20,29,31,33 §2.2: 3,5,9,12

Hand In

6) Find a general solution for the differential equations:
a) $y' + \sin(t)y = 0$
b) $y' + 2y = -t + e^{4t}$

7) Solve the initial value problem:

$$y' + 5y = \sin(t), \qquad y(0) = 1$$

8) Determine the general solution of the differential equation

$$2\frac{\mathrm{d}y}{\mathrm{d}t} = y^2 \cdot \sin^2(t)$$

9) Consider the differential equation:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x + x^3}{y}.$$

a) Solve the initial value problem for $y(\frac{1}{2}) = 1$
b) Solve the initial value problem for $y(\frac{1}{2}) = -1$
c) For which x values does the initial value problem $y(2) = 2$ have a solution?

10) Solve the Bernoulli equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = 2y - 4y^2$$

11) (Problems 35,36 from §2.1:)

Variation of Parameters. Consider the following method of solving the general linear equation of first order:

$$y' + p(t)y = g(t). \tag{i}$$

(a) If $g(t)$ is identically zero, show that the solution is

$$y = A \exp\left[-\int p(t)\,dt\right], \tag{ii}$$

where A is a constant.

(b) If $g(t)$ is not identically zero, assume that the solution is of the form

$$y = A(t) \exp\left[-\int p(t)\,dt\right], \tag{iii}$$

where A is now a function of t. By substituting for y in the given differential equation, show that $A(t)$ must satisfy the condition

$$A'(t) = g(t) \exp\left[\int p(t)\,dt\right]. \tag{iv}$$

(c) Find $A(t)$ from Eq. (iv). Then substitute for $A(t)$ in Eq. (iii) and determine y. This technique is known as the method of **variation of parameters**; it is discussed in detail in Section 3.7 in connection with second order linear equations.

Solve the differential equation $y' - 2y = t^2 e^{2t}$ using this method.