Practice

§9.1: 5,9,13
§9.2: 5,7,14
§9.3: 7,9,11
§9.5: 1,5,11

Hand In

(You may use MAPLE (or similar) to compute eigenvalues and eigenvectors. However you may not use the `MatrixExponential` function (or similar) to compute $\exp(At)$ without any further work.

59*) Solve the following initial value problem:

$$\begin{aligned} \frac{\mathrm{d}^2x}{\mathrm{d}t^2} &= 4\frac{\mathrm{d}x}{\mathrm{d}t} - x(t) + 6y(t), \qquad && x(0)=1, x'(0)=0 \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= -x(t), && y(0)=2 \end{aligned}$$

60*) Let

$$A = \begin{pmatrix} 18 & 3 & 38 \\ 177 & 41 & 473 \\ -19 & -4 & -47 \end{pmatrix} = M \cdot \begin{pmatrix} 4 & 1 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{pmatrix} \cdot M^{-1} \qquad \text{with} \quad M = \begin{pmatrix} 1 & 2 & -2 \\ 8 & 29 & -28 \\ -1 & -3 & 3 \end{pmatrix}$$

a) Calculate $\exp(Jt)$ and $\exp(At)$
b) Determine a fundamental set of solutions for the system of differential equations $\underline{\mathbf{x}}' = A \cdot \underline{\mathbf{x}}$.
c) Determine **one** particular solution for the system of differential equations

$$\underline{\mathbf{x}}' = A \cdot \underline{\mathbf{x}} + \begin{pmatrix} 20\cos^2(t) + 12 - 32t \\ 160\cos^2(t) + 174 - 448t \\ -20\cos^2(t) - 18 + 48t \end{pmatrix}$$

61*) Classify the singularity at $(0,0)$ for the system

$$\underline{\mathbf{x}}' = \begin{pmatrix} 1 & -2 \\ 3 & a \end{pmatrix} \cdot \underline{\mathbf{x}}$$

depending on the (real) value of a.
Hint: First classify for which values of a the characteristic polynomial has multiple, and real or complex roots. Then determine the respective values of the roots.

62*) Consider the following system of differential equations:

$$\frac{dx}{dt} = (2+x)(y-x), \quad \frac{dy}{dt} = y(2+x+x^2)$$

a) determine the critical points
b) (MAPLE) sketch the direction field
c) Determine which critical points are asymptotically stable or unstable, and classify their types.

63*) Consider the following system of differential equations:

$$\begin{aligned} x' &= -10x+10y \\ y' &= 28x-y-xz \\ z' &= xy-8z/3 \end{aligned}$$

a) Determine the critical points of this system (there are 3).
b) Calculate the Jacobi matrix.
c) Consider the linearized problems at the critical points. Determine their stability.

Problems marked with a *are bonus problems for extra credit.