Mathematics 340-3 **Homework (due Nov. 18)** A. Hulpke

Practice

§7.3: 1,8,12,15,21,25

Hand In

50) Consider the system of differential equations

$$\begin{aligned} x_1' &= 6x_1 - 1x_2 \\ x_2' &= 5x_2 \end{aligned}$$

a) Using MAPLE, sketch the direction field for this system.
b) Using this direction field, determine a new set of variables z_1, z_2 that will decouple the equations and use these to solve the system.

51) Let $A = \begin{pmatrix} 4 & -6 \\ 1 & -1 \end{pmatrix}$.
a) Compute the eigenvalues and corresponding eigenvectors of A.
b) Determine a general solution to the following system of differential equations:

$$\begin{aligned} x_1' &= 4x_1 - 6x_2 \\ x_2' &= 1x_1 - 1x_2 \end{aligned}$$

Note: After Problem 51 I assume you know how to calculate eigenvalues and eigenvectors by hand. From now on you are welcome to calculate these using (for example) MAPLE. Note however that you might have to calculate eigenvectors by hand in the final.

52) Consider the following system of differential equations $\underline{\mathbf{x}}'(t) = A \cdot \underline{\mathbf{x}}(t)$ with

$$A = \begin{pmatrix} 47 & 0 & -200 \\ -20 & -3 & 80 \\ 10 & 0 & -43 \end{pmatrix}.$$

a) Determine eigenvalues and eigenvectors for A.
b) Determine $\exp(A \cdot t)$.
c) Determine a solution to the initial value problem for $\underline{\mathbf{x}}(0) = (1,2,3)^T$.

53*) Let $A \in \mathbb{R}^{n\times n}$ be a matrix. Show that A is singular (i.e. not invertible) if and only if 0 is an eigenvalue of A.

54) Consider the initial value problem

$$\underline{\mathbf{x}}'(t) = \begin{pmatrix} 10 & -53 \\ 10 & 12 \end{pmatrix} \cdot \underline{\mathbf{x}}(t), \quad \underline{\mathbf{x}}(0) = \begin{pmatrix} 23 \\ 23 \end{pmatrix}.$$

Problems marked with a *are bonus problems for extra credit.