

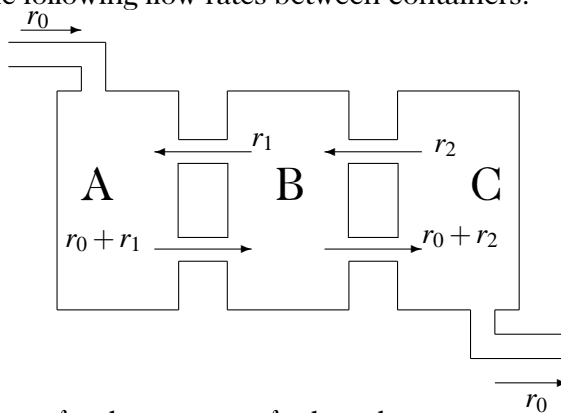
Practice

§7.1: 1,3,4,5,6,15
 §7.2: 1,2,21,23,26

Hand In

46) Suppose you have three water containers, each containing a volume V of water, connected as in the picture. Water, containing salt at a concentration c_0 is flowing into container A at a rate of r_0 per minute. Furthermore we have the following flow rates between containers:

From	To	rate
A	B	$r_0 + r_1$
B	A	r_1
B	C	$r_0 + r_2$
C	B	r_2
C	out	r_0



Set up a system of differential equations for the amount of salt in the containers.

47) Let $y(t)$ be a function fulfilling a second order linear differential equation initial value problem:

$$y'' + b \cdot y' + c \cdot y = g(t), \quad y(t_0) = y_0, y'(t_0) = y'_0$$

We set $w(t) := y'(t)$. Write the above differential equation as a system of two **first** order differential equations for y and w .

48) For which values of λ is the following matrix invertible?

$$\begin{pmatrix} 2 & -\lambda & 2 \\ 1 & 1 & \lambda \\ -2 & 1 & -2 \end{pmatrix}$$

49) We define the exponential function of a matrix A (in the same way as for numbers) as

$$\exp(A) := \sum_{i=0}^{\infty} \frac{1}{i!} A^i.$$

Using this definition, calculate

$$\exp \begin{pmatrix} 2t & 0 \\ 0 & -5t \end{pmatrix} \quad \text{and} \quad \exp \begin{pmatrix} t & 1 \\ 0 & t \end{pmatrix}$$