Practice

§6.4: 3,5,7,11
§6.5: 1,5,6,8

Hand In

42) Solve the initial value problem

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2}+4y=g(t)=\begin{cases}0 & t<2\\ -1 & 2\le t\end{cases},\qquad y(0)=y'(0)=0$$

43) Using Laplace-transforms, find the solution to

$$t'=\delta(t), y(0_-)=0$$

What interpretation of δ does this imply?

44) Suppose that an undampened spring-mass system with mass m, spring constant k and natural frequency $\omega=\sqrt{k/m}$ is initially in motion with position $y(0)=0$ and velocity $y'(0)=v_0=1$. At a later time $t_1=\pi/\omega$ the mass is struck with a hammer imparting an instantaneous transfer of momentum $\Delta\mu=mv_0=m\cdot 1$.
The motion of the mass may be modelled by

$$y''+ky=\Delta\mu\cdot\delta(t-t_1),\quad y(0)=0, y'(0)=1$$

Solve the differential equation, and describe the effect of the strike on the movement.

45*) Show that there cannot be a function $f(t)\colon \mathbb{R}\to\mathbb{R}$ such that $\int_{-\infty}^{\infty} f(t)g(t)\,\mathrm{d}t=g(0)$ for all functions $g\colon \mathbb{R}\to\mathbb{R}$. (Thus the DIRAC-$\delta$ distribution is not a proper function.)

Problems marked with a *are bonus problems for extra credit.