Mathematics 340-3 **Homework (due Sep. 2)** A. Hulpke

Practice

§1.1:7,15,16
§1.2:3,6,11
§1.3: 2,3,4,9,10,12,17,20

Hand In

1) For the following problems, find the order of the differential equation and state whether it is linear or nonlinear:

$$(1+y^2)\frac{\mathrm{d}^2y}{\mathrm{d}t^2}+t\frac{\mathrm{d}y}{\mathrm{d}t}+y=e^t \tag{1}$$

$$\frac{\mathrm{d}^4y}{\mathrm{d}t^4}+\frac{\mathrm{d}^3y}{\mathrm{d}t^3}+\frac{\mathrm{d}^2y}{\mathrm{d}t^2}+\frac{\mathrm{d}y}{\mathrm{d}t}+y=1 \tag{2}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t}+ty^2=0 \tag{3}$$

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2}+\sin(t+y)=\sin(t) \tag{4}$$

$$\tag{5}$$

2) Verify that the given functions are solutions of the differential equation:
a) $ty'-y=t^2$, $y=3t+t^2$
b) $t^2y''+5ty'+4y=0$, $t>0$, $y_1(t)=t^{-2}$, $y_2(t)=t^{-2}\ln(t)$.
c) $y'-2ty=1$, $y=e^{t^2}\displaystyle\int_0^t e^{-s^2}\,\mathrm{d}s+e^{t^2}$

3) Determine the value r for which the following differential equation has a solution of the form $y=e^{rt}$:

$$y''+y'-6y=0$$

4) Determine a solution for the initial value problem:

$$\frac{\mathrm{d}y}{\mathrm{d}t}=3y+4,\qquad y(0)=10$$

5) A student has bougth a coffee at Sweet Sinsations just a few minutes before the lecture starts (to which – it taking place in a computer lab – no coffee can be brought).
As the coffee is still very hot, the student would like to have it as cool as possible in 2 minutes and she deliberates whether she should put in the milk immediately. Fortunately

she took a course in differential equations.
Assume: There are c units of coffee at initial temperature $90°C$ and $m = \frac{1}{10} \cdot c$ units of milk at fridge temperature $10°C$. The room temperature is $20°C$.
For the purpose of this problem assume that both water and coffee have the same specific heat: The temperature after mixing is $\frac{c \cdot T_{coff} + m \cdot T_{milk}}{c+m}$. Also both coffee and coffee/milk mixture cool down with coefficient $0.1\frac{1}{sec}$. We also assume that the milk is in the fridge and does not heat up before mixing.
a) What is the temperature when the coffee cools down for t seconds and then the milk is mixed in?
b) What is the temperature after t seconds when the milk is mixed in immediately at the start?
c) Should one mix in the milk first, or later?