17 **17** Selecting n balls out of $n+n = 2n$ options has $\binom{2n}{n}$ possibilities.

~~OTOH, if we separate according to the number of green balls~~, ~~say the green balls~~, a ~~selection with k green balls gives~~ $\binom{n}{k}$ options for the green balls and $\binom{n}{n-k} = \binom{n}{k}$ ~~options for the gold balls, thus~~ $\binom{n}{k}^2$ options in total. Thus $\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$.

18 done in class

19 Let $n \geq n_0 = 100$. Then $3n \leq n^5$ and $17 \leq n^5$.

Thus $n^5 + 3n + 17 \leq n^5 + n^5 + n^5 = 3 \cdot n^5$

~~thus (with $C = 3$) $n^5 + 3n + 17 = O(n^5)$~~

20 $\frac{2}{n}$, $\sqrt{n}$, $\frac{n}{n+2}$, ~~$n \log \log n$, $n \log n$ equal to $n \log n^2$~~,

$n \log^2 n$, $n^2 \log n$, n^3, 2^n

21 There are $9!$ permutations in total.

~~Let A_i = permutations that fix i~~

Then $|A_i| = (9-1)! = 8!$ and for $I \subseteq \{1..9\}$

$\left|\bigcap_{i\in I} A_i\right| = $ # permutations fixing all pts in I

$= (9-|I|)!$ Thus by PIE

$\#\text{derangements} = 9! - \left|\bigcup A_i\right|$

$= 9! - \sum_{\substack{I\subseteq\{1..9\}\\ |I|\geq 1}} (-1)^{|I|} \left|\bigcap_{i\in I} A_i\right|$

$= 9! \sum_{k=1}^{9} \sum_{\substack{I\subseteq\{1..9\}\\ |I|=k}} (-1)^k (9-k)!$

$= 9! - \sum_{k=1}^{9} (-1)^k (9-k)! \binom{9}{k}$

Would be value for $k=0$

$= \sum_{k=0}^{9} (-1)^k (9-k)! \binom{9}{k} = \sum_{k=0}^{9} (-1)^k \binom{9}{k} k!$

replace k with $9-k$

22 done in class