

11] There are two O, two L. Thus 4 distinct cases.

a) at most one O, one L:

5 letters out of 9 possibilities: $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5$

b) two O, at most one L:

$\binom{5}{2} \cdot 8 \cdot 7 \cdot 6$ (There are 8 non-O letters)
 $\uparrow$ 3 non-O positions
positions of O's

c) same for two L, at most one O

d) two O, two L:

$\binom{5}{2} \binom{3}{2} \cdot 7$ remaining spaces after O-choice
 $\uparrow \quad \uparrow \quad \uparrow$
O-pos L-pos # of non O/L letters.

$$\text{Thus: } 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 + 2 \cdot \binom{5}{2} \cdot 8 \cdot 7 \cdot 6 + \binom{5}{2} \binom{3}{2} \cdot 7 = 22050$$

12) a) 36 symbols : 36^8

b) $\binom{8}{1} \cdot 10 \cdot 26^7$ rest
↑
position digit #digits

c) Count total - forbidden:

$$36^8 - \underbrace{26^8}_{\text{no digit}} - \underbrace{10^8}_{\text{no letter}}.$$

13 a) $\binom{100}{10}$

b) $\binom{100}{10} \cdot 10$ (choose committee, then chair)

or $\binom{100}{1} \cdot \binom{99}{9}$ (choose chair first)

c) $100 \cdot 99 \cdots 91 = \frac{100!}{90!}$

14) Need on more than 5 fresh + 4 seph...

i.e. $1 + (5+4+3+2) = 15$

15]

If we do not distinguish different
elves/fellies: 8 possible positions
for felines: OEOEOEOE...OEO

thus $\binom{8}{5}$.

If the elves, felines have individuality:

$$\binom{8}{5} \cdot 8! \cdot 5!$$

$$B) \binom{14}{4} = 1001 \text{ and } \binom{k}{4} < 601 \text{ for } k \leq 14$$

So 14 species.