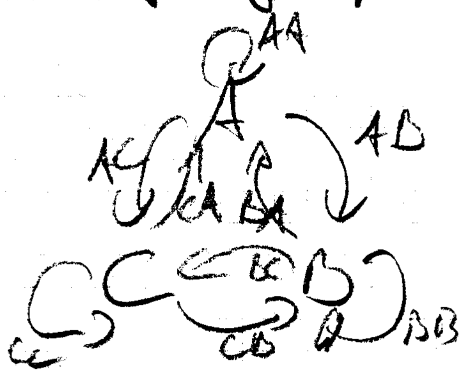
46| De Bruijn graph



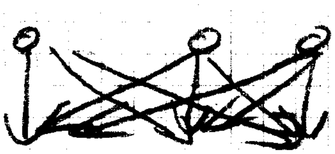
AABCACCBB

47| Done in class

48| Let $v \in V$ and set $I := \{u \mid (u,v) \in E\}$, $O := \{u \mid (v,u) \in E\}$. By definition of tournament $V = \{v\} \cup I \cup O$ all the sets are disjoint. By induction on $|V|$ (base case is trivial) there is a directed path through all vertices of the directed graph Γ_I, the induced subgraph on I as well as on Γ_O. Combine the paths.

49| Done in class

50| Yes, if the number of students are a multiple of 6 and we can partition the graph into components that are a directed version of $K_{3,3}$:



51: Let G be bipartite with vertex sets V_1, V_2.

Let $(v_1, v_2, v_3, \ldots, v_k, v_1)$ be a cycle

WLOG $v_1 \in V_1$. Then (bipartite!) $v_2 \in V_2$,

$v_3 \in V_1$, $\ldots$ $v_i \in \begin{cases} V_1 & i \text{ even} \\ V_2 & i \text{ odd} \end{cases}$

As $v_{k+1} = v_1$, we need $v_k \in V_2$, so k is even.

Vice versa, assume that all cycles have even length.
Pick a vertex $v \in U$ and let $V_1 = \{x \in U \mid d(v,x) \text{ even}\}$
$V_2 = \{x \in U \mid d(v,x) \text{ odd}\}$. Then no vertices
$a, b \in V_2$ can be connected as $(\underbrace{x, \ldots, a}_{\text{odd}}, \underbrace{b, \ldots, x}_{\text{odd}})$
(using the paths for distance)
is a cycle of odd length ↯ (The cycle is $(x, \ldots, a, b, \ldots, x)$)
Ditto for $a, b \in V_1$. Thus V_1, V_2 is a bipartition.