HW1 due 8/30/17

1) 1) $\{3,5\}$

2, 4) $\{2,4,6,1,7,9\}$
(those in one set but not the other)

3) A and B share only 3, 5. So pairs must start and end with 3, 5:

$\{(3,3),(3,5),(5,3),(5,5)\}$

5) $\{2,3\}$

6) $\{3,5,7,9,11,13,15,17,19,21\}$

7) $\{(2,1),(2,3),(2,5),(3,1),(3,3),(4,1),(4,3),(5,1),(6,1)\}$

2) (This is easiest by just summing the 6 or 12 terms each time)

1) 3 2) 78 3) 9 4) 180

3) Left side: The inner sum runs from 1 to i, that is we sum over those pairs indexed by (i,j) such that $j \leq i$

Right side: The inner sum runs $i=j$ to n, that is we sum over pairs (i,j) where $i \geq j$. This is the same set of pairs as the LHS.

4) <u>Note</u>: When I write you should write the text of the full proof,

this does mean that all should be written, not just the gaps filled.

Proof: We prove the statement by induction.

Base case: Let $n=1$. Then

$$\sum_{i=1}^{1} i^2 = 1^2 = 1 = \frac{1\cdot 2\cdot 3}{6}$$ and the statement holds.

Inductive Step: Let $n>1$ and assume that the statement is true for $n-1$.

Then $$\sum_{i=1}^{n} i^2 = \sum_{i=1}^{n-1} i^2 + n^2$$

By induction we know that the first summand is equal to

$$\sum_{i=1}^{n-1} i^2 = \frac{(n-1)(n-1+1)(2(n-1)+1)}{6}.$$

$$= \frac{2n^3-3n^2+n}{6}$$ and therefore

$$\sum_{i=1}^{n} i^2 = \frac{2n^3-3n^2+n+6n^2}{6} = \frac{2n^3+3n^2+n}{6}$$

$$= \frac{n(n+1)(2n+1)}{6}.$$ The statement thus holds for $n+1$. By the principle of induction the statement thus holds for any positive n, as was to be shown.

5 was done in class.