Ex: Taylor pol for $f(x) = (x+1)^{5/2}$ around $a=0$

deg 2

$$\sum_{i=0}^{\deg} \frac{f^{(i)}(0)}{i!} x^i$$

$f(0) = 1$.

$f'(x) = 5/2\,(x+1)^{3/2}$ | $f'(0) = 5/2$

$f''(x) = \frac{15}{4}(x+1)^{1/2}$ | $= \frac{15}{4}$

$$\Rightarrow \frac{1}{0!}x^0 + \frac{\frac{5}{2}}{1!}x^1 + \frac{\frac{15}{4}}{2!}x^2$$

$$= 1 + 5/2\,x + \frac{15}{8}x^2$$

Ex: Taylor series for $\frac{1}{1-x^5}$

$$= 1 + x^5 + x^{10} + x^{15} + x^{20} + \cdots$$

$$= \sum_{i=0}^{\infty} x^{5i}$$

Geom Series

$$1 + x + x^2 + \cdots = \frac{1}{1-x}$$

x^5

Know $\dfrac{1}{1-x^5} = \sum_{i=0}^{\infty} x^{5i}$ $\Big\} \frac{d}{dx}$

$$\frac{5x^4}{(1-x^5)^2} = -\frac{1}{(1-x^5)^2}(-5x^4) \quad \Big| \quad \sum_{i=1}^{\infty} (5i)\, x^{(5i-1)}$$

x^4 $\qquad$ $x \cdot \lg(x) = O(x^4)$

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Show: L'Hospital:

$$\lim_{x\to\infty} \frac{x \cdot \lg(x)}{\cancel{x^4}\,{}^3} = \lim_{x\to\infty} \frac{1/x}{3x^2} = 0$$

$$" \frac{1}{3x^2}$$

$$\int \log(x) \cdot x^4 \, dx = \frac{1}{5} \log(x) \cdot x^5 - \int \frac{1}{5} x^5 \cdot \frac{1}{x} \, dx$$

$$\frac{x^4}{5}$$

$f(x) = \log(x)$, $g'(x) = x^4$

$f'(x) = 1/x$, $g(x) = \frac{1}{5} x^5$

$$= \frac{1}{5} \lg(x) x^5 - \frac{x^5}{25} + C$$

$$\int \frac{\sin(x)}{x} \, dx = x - \frac{x^3}{3 \cdot 3!} + \frac{x^5}{5 \cdot 5!} - \frac{x^7}{7 \cdot 7!} + \cdots + C$$

Taylor: $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

$$\frac{\sin(x)}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5} - \frac{x^6}{7!} + \cdots$$

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