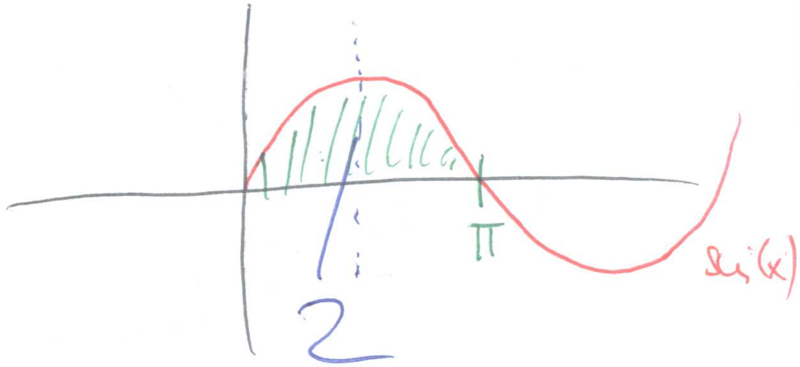
Ex:

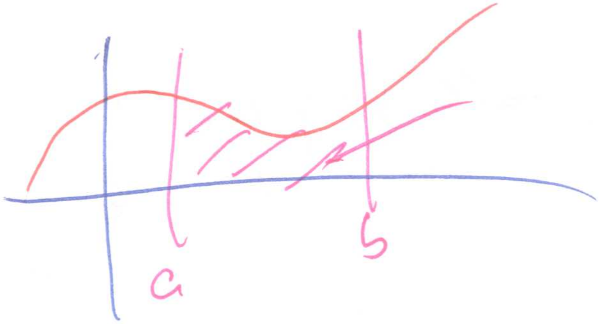
$$\int_0^{\pi} \sin(x)\,dx = \left(-\cos(x)\right)\Big|_0^{\pi}$$

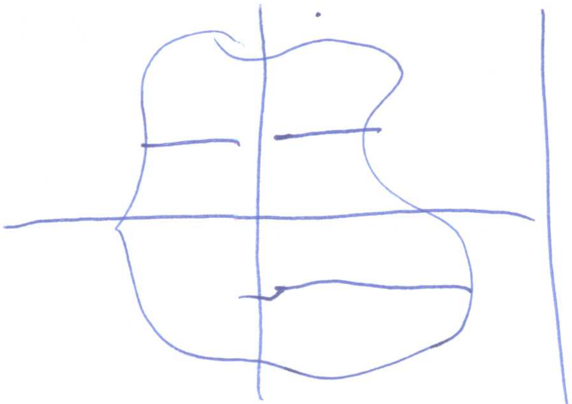
$$= \underbrace{-\cos(\pi)}_{--1} + \underbrace{\cos(0)}_{+1} = 2$$



$$A = \int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

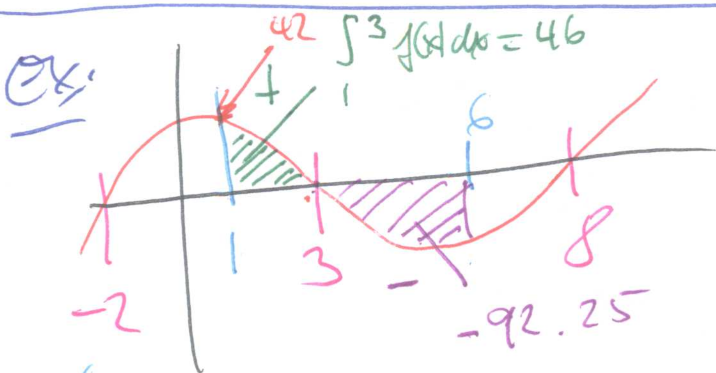
$$\frac{d}{dx}F(x) = f(x), \quad F(x) = \int f(x)\,dx$$





Ex:

$$f(x) = x^3 - 9x^2 + 2x + 48$$

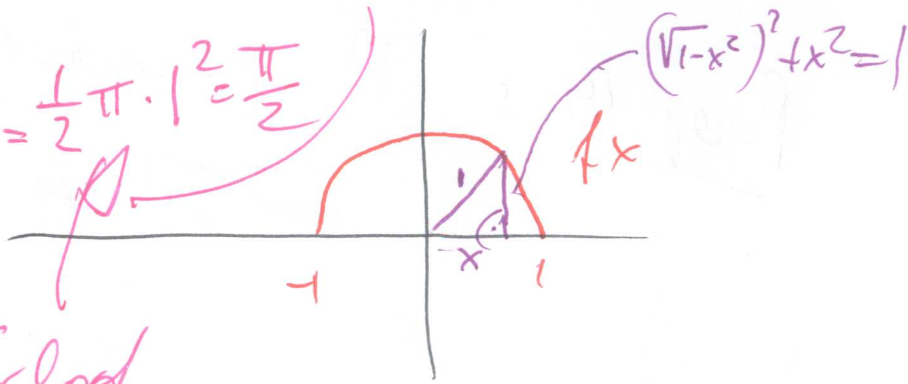


$$\int_1^6 x^3 - 9x^2 + 2x + 48\,dx = \frac{1}{4}x^4 - 3x^3 + x^2 + 48x \Big|$$

$$= -46.25$$

Ex: $\int_{-1}^{1}\sqrt{1-x^2}\,dx$

Area under half circle radius 1 $= \frac{1}{2}\pi\cdot 1^2 = \frac{\pi}{2}$



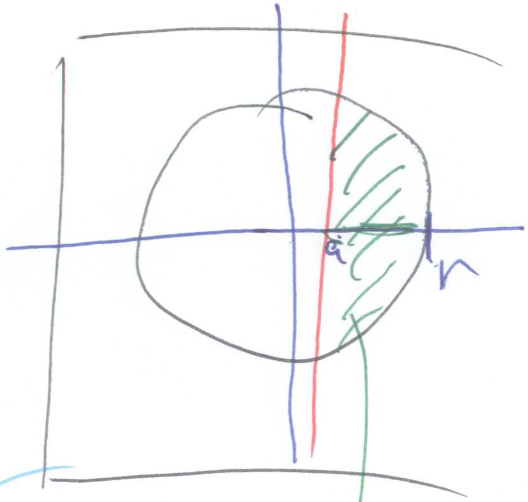
Antider. of $\sqrt{1-x^2}$

Computer: $\frac{1}{2}\left(x\sqrt{1-x^2} + \arcsin(x)\right) + C$

School geometry.

Verify: $\frac{d}{dx} = \frac{1}{2}\left(\sqrt{1-x^2} - x\frac{1\cancel{2}x}{\cancel{2}\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}}\right)$

$= \frac{1}{2}\left(\sqrt{1-x^2} + \frac{-x^2+1}{\sqrt{1-x^2}}\right) = \frac{1}{2}\left(\sqrt{1-x^2} + \sqrt{1-x^2}\right) = \sqrt{1-x^2}$

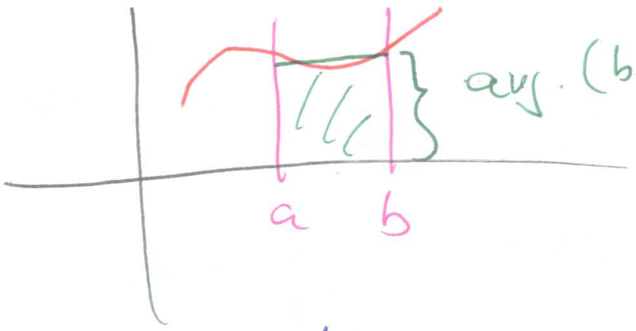


$\frac{1}{2}\left(x\sqrt{1-x^2} + \arcsin(x)\right)\Big|_{-1}^{1} = \frac{1}{2}\frac{\pi}{2} + \frac{1}{2}\frac{\pi}{2} = \frac{\pi}{2}$

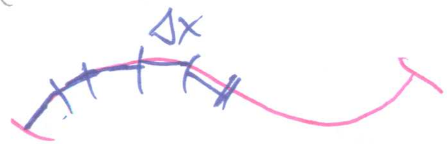
$= 2\cdot\int_{a}^{r}\sqrt{r^2-x^2}\,dx$

Uses:

• Averaging



• Compute summations of infinitely small things.

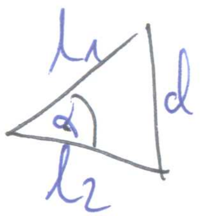


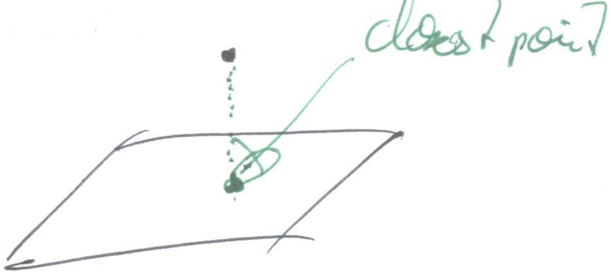
how long?

$$\lim_{\Delta x \to 0} \sum \rightsquigarrow \int (\cdots) \, dx$$

Segments length 0

• Geometry on functions.





"Length of fct."

$$\sqrt{\int_a^b f(x)^2 \, dx}$$

definition

depend on problem

length of line from $(0,0,0)$ to (a,b,c)

$$\sqrt{a^2+b^2+c^2} = \sqrt{\sum x_i^2}$$

$$\int \exp(\sin(x)) \cdot \boxed{\cos(x)\, dx} = \int \exp(u)\, \boxed{du} = \exp(u) + C$$

$\boxed{u = \sin(x)}$ } calc du

$$= \exp(\sin(x)) + C$$

$$\frac{du}{dx} = \cos(x) \rightarrow dx = \frac{du}{\cos(x)}$$

$$\int \exp(\sin(x)) \cdot \sin(x) \cdot \boxed{\cos(x)\, dx} = \int \exp(u)\, u \cdot du$$

$$\int \exp(\sin(x)) \cdot x \cdot \cos(x)\, dx = \int \exp(u) \arcsin(u)\, du$$

Missing cos(x)

$$\int \exp(\sin(x))\, dx =$$

$$\int \log(x)\cdot x \; dx = \frac{1}{2}x^2\cdot \log(x) - \int \frac{1}{x}\cdot\frac{1}{2}x^2 \, dx$$

$$\frac{1}{2}x$$

$f(x) = \log(x)$, $g'(x) = x$

$f'(x) = \frac{1}{x}$, $g(x) = \frac{1}{2}x^2$

$$= \frac{1}{2}x^2\log(x) - \frac{1}{4}x^2 + C$$