Note: Our limit criterion can be unsatisfactory

ex: $a_1 = 1$

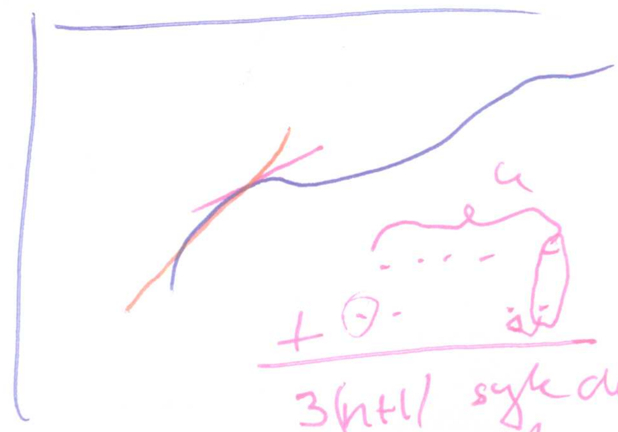
$a_{i+1} = 1 + \frac{1}{a_i} \qquad (i \ge 1)$

$a_2 = \frac{3}{2},\ a_3 = \frac{5}{3},\ a_4 = \frac{8}{5},\ \ldots \rightsquigarrow 1.618033\ldots$

satisfies $x = 1 + \frac{1}{x}$

Theorem: A sequence that is increasing (decreasing) and bounded from above (below) has a limit $\in \mathbb{R}$

$3,\ 3.1,\ 3.14,\ \ldots \longrightarrow \pi$



$3(n+1)$ single digit additions

Note: (a_i), (b_i) are sequences

Form new sequences $c_i = a_i + b_i$

$d_i = a_i \cdot b_i$

Rem: $\lim\limits_{i\to\infty} (a_i + b_i) = \lim\limits_{i\to\infty} a_i + \lim\limits_{i\to\infty} b_i$ If both limits exist & are numbers

$\cdot \qquad\qquad \cdot$

$- \qquad\qquad -$

$/ \qquad\qquad / \leftarrow$ as long as b_i, $\lim b_i \neq 0$

Ex: $a_i = 3 + \frac{1}{i}$ (Limit: $\lim\limits_{i\to\infty} a_i = 3$

$b_i = 5 - \frac{1}{i^2}$ $\lim\limits_{i\to\infty} b_i = 5$

$a_i + b_i = 3 + 5 + \frac{1}{i} - \frac{1}{i^2}$

$= 8 + \frac{i - 1}{i^2}$ has limit 8

Product: $\left(3 + \frac{1}{i}\right)\left(5 - \frac{1}{i^2}\right) = 15 + \frac{5}{i} - \frac{3}{i^2} - \frac{1}{i^3}$

Series:

$a_1, a_2, a_3, \ldots$, $\lim_{i \to \infty} a_i = 0$

Series: $a_1 + a_2 + a_3 + \cdots = ?$

The infinite sum is the limit of partial sums $a_1,\ a_1+a_2,\ a_1+a_2+a_3, \ldots$

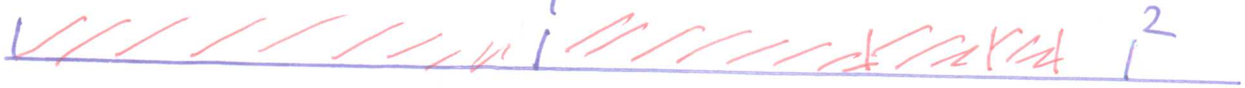
$$\sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n$$

get sequence of partial sums $b_n = \sum_{i=1}^{n} a_i$

The infinite sum $\sum_{i=1}^{\infty} a_i = \lim_{n \to \infty} \sum_{i=1}^{n} a_i$ if limit exists

sometimes 0

Ex: $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = \sum_{i=0}^{\infty} \frac{1}{2^i} = 2$



But: $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\cdots = \sum_{i=1}^{\infty} \frac{1}{i} = \infty$

"harmonic series"

as $\underbrace{1}_{\substack{1 \\ \text{1 entry,} \\ > \frac{1}{2}}} + \underbrace{\frac{1}{2}+\frac{1}{3}}_{\substack{\text{2 entries} \\ > \frac{1}{4}, \\ \text{sum} \\ > \frac{2}{4} = \frac{1}{2}}} + \underbrace{\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}}_{\substack{\text{4 entries} \\ > \frac{1}{8} \\ \text{sum} \\ > \frac{4}{8} = \frac{1}{2}}} + \underbrace{\frac{1}{8}+\cdots}_{8}$

$> \frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\cdots$

$= \infty$

Geometric Series: Start value a $\overset{= a\cdot r^0}{a}, a\cdot r, a\cdot r^2, \ldots$

Factor r

Sum over such a sequence. $\boxed{\sum_{i=0}^{n} a\cdot r^i}$

Ex: \$100 in account, 3% interest.

Interest weight: Year 1: $100 \cdot 0.03$, \$103 ; Year 2: $\underline{103 \cdot 0.03}$

$(\underline{100+100\cdot 0.03})\cdot 0.03$