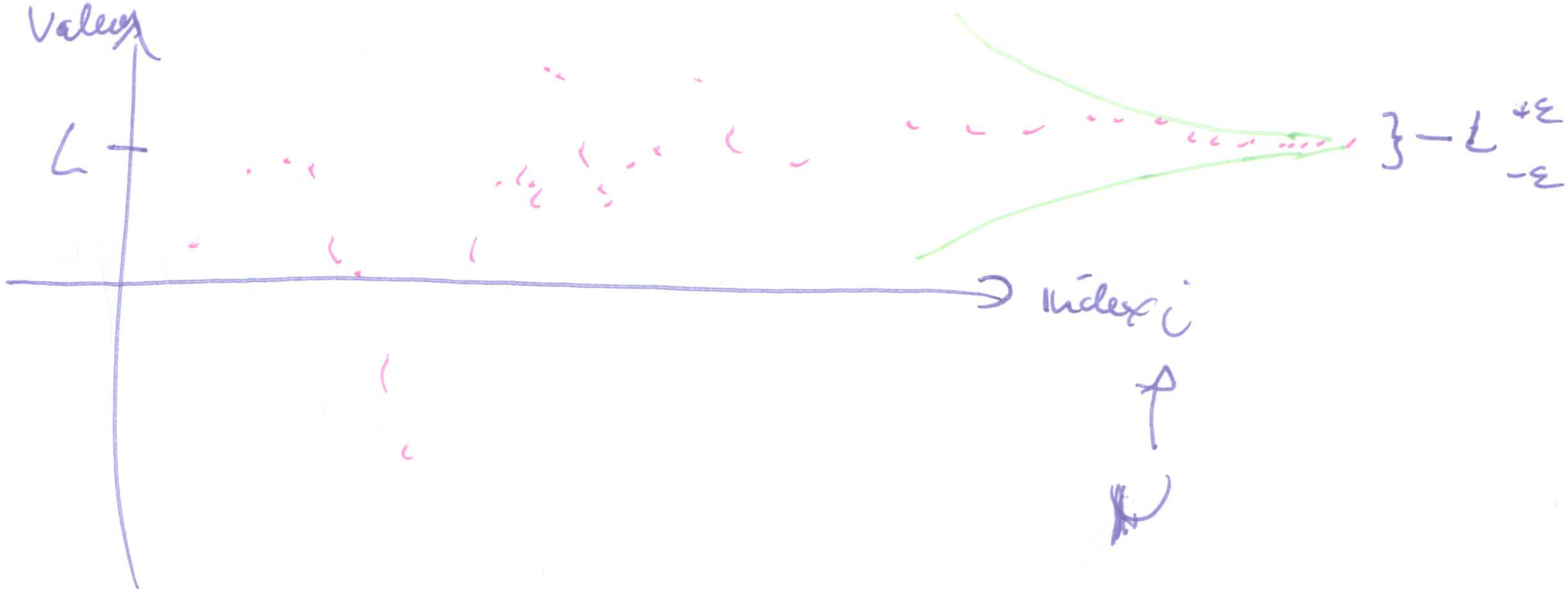


Def.: A sequence (a_i) converges to lim L (write: $\lim\limits_{i\to\infty} a_i = L$)

if For all $\varepsilon > 0$ ex. N: If $i \geq N$: $|a_i - L| \leq \varepsilon$

or $<$

Ex: $a_i = 5 + \frac{1}{i}$ $\quad L = \lim\limits_{i\to\infty} a_i = 5$

Proof: Let $\varepsilon > 0$. Set $N = \frac{1}{\varepsilon}$.

Then for $i \geq N$: $|a_i - L| = |5 + \frac{1}{i} - 5| = |\frac{1}{i}| = \frac{1}{i}$

$\leq \frac{1}{N} = \frac{1}{1/\varepsilon} = \varepsilon$. Thus $\lim\limits_{i\to\infty} a_i = 5$

Scratch Paper

$\varepsilon \geq |a_i - L| = |\cancel{5} + \frac{1}{i} - \cancel{5}|$

$= |\frac{1}{i}| = \frac{1}{i}$. Solve for index!

$\Rightarrow i \geq \boxed{\frac{1}{\varepsilon}}$ Value for N

Ex: $a_i = \frac{4i+3}{7i-1}$ | Claim: $\lim_{i \to \infty} a_i = \frac{4}{7} = L$

Proof: Let $\varepsilon > 0$. Set $N = \frac{1}{49}\left(\frac{25}{\varepsilon}+7\right)$

Then for $i \geq N$: $|a_i - L| = \left|\frac{4i+3}{7i-1} - \frac{4}{7}\right|$

$$= \left|\frac{28i+21-28i+4}{49i-7}\right| = \frac{25}{49i-7} \leq \frac{25}{49N-7} = \frac{25}{\cancel{49}\cdot\frac{1}{\cancel{49}}\left(\frac{25}{\varepsilon}+7\right)-7}$$

$$= \frac{\cancel{25}}{\left(\frac{\cancel{25}}{\varepsilon}\cancel{+7}\right)\cancel{-7}} = \frac{1}{1/\varepsilon} = \varepsilon .$$

We prove that the sequence a_i has the limit L

Scratch:

$$\varepsilon \geq |a_i - L| = \left|\frac{4i+3}{7i-1} - \frac{4}{7}\right|$$

$$= \left|\frac{\cancel{28i}+21-\cancel{28i}+4}{49i-7}\right|$$

$$= \left|\frac{25}{49i-7}\right| = \frac{25}{49i-7}$$

Solve for i:

$$\Leftrightarrow 49i - 7 \geq \frac{25}{\varepsilon}$$

$$\Leftrightarrow 49i \geq \frac{25}{\varepsilon} + 7$$

$$\Leftrightarrow i \geq \boxed{\frac{1}{49}\cdot\left(\frac{25}{\varepsilon}+7\right)}$$

Ex: show that the seq. $a_i = 3 + \frac{1}{i^2}$ has limit $L = 3$

Proof. Let $\varepsilon > 0$. Set $N = \frac{1}{\sqrt{\varepsilon}}$. Then for $i \geq N$:

$$|a_i - L| = \left|3 + \frac{1}{i^2} - 3\right| = \left|\frac{1}{i^2}\right| = \frac{1}{i^2} \leq \frac{1}{N^2} = \frac{1}{\left(\frac{1}{\sqrt{\varepsilon}}\right)^2}$$

replace i by N

$$= \frac{1}{1/\varepsilon} = \varepsilon.$$

$$\varepsilon \overset{!}{\geq} |a_i - L| = \left|\cancel{3} + \frac{1}{i^2} - \cancel{3}\right| = \left|\frac{1}{i^2}\right| = \frac{1}{i^2}$$

Solve for i

$$\Leftrightarrow i^2 \geq \frac{1}{\varepsilon}$$

$$\Leftrightarrow i \geq \frac{1}{\sqrt{\varepsilon}}$$

$$\varepsilon \geq \frac{1}{i^2} \quad \Big| \cdot i^2,\ \div \varepsilon$$

$$\Leftrightarrow i^2 \geq \frac{1}{\varepsilon}$$

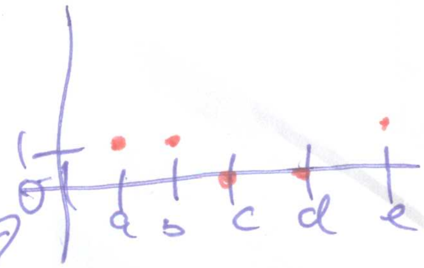
(Sect III.6

$|A| = 5$, $|B| = 2$

Fcts from A to B: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$

choices of image for first

img 2nd



All ranges

$a \mapsto 1$

$b \mapsto 1$

$c \mapsto 0$

$d \mapsto 0$

$e \mapsto 1$