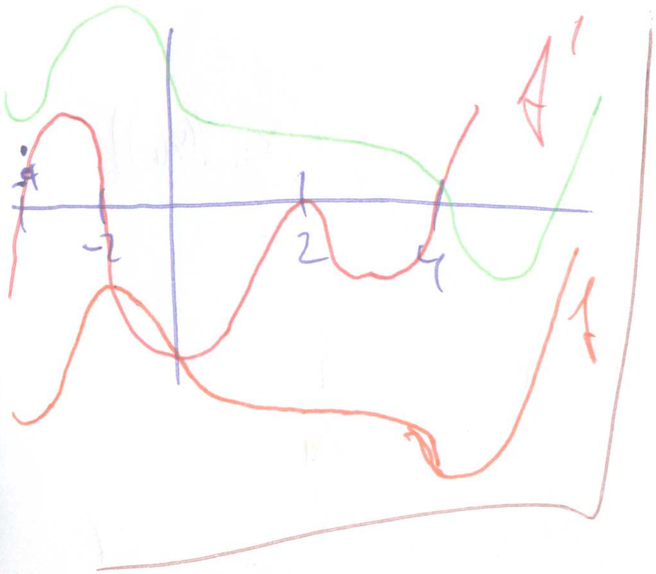
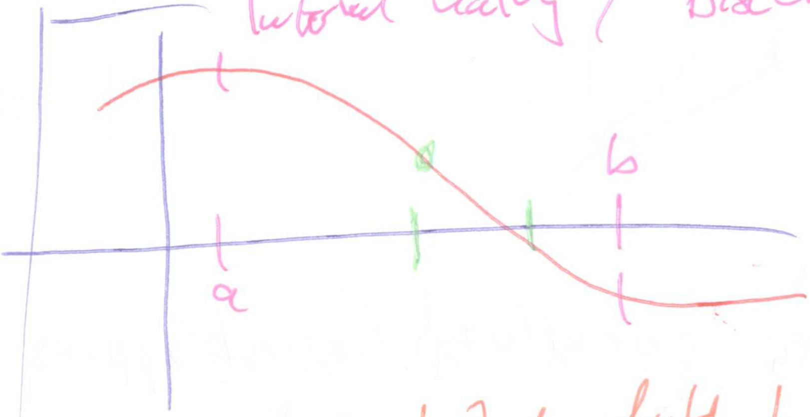


Newton's method

Given f, find x_0 : $f(x_0)=0$

Interval halving / bisection

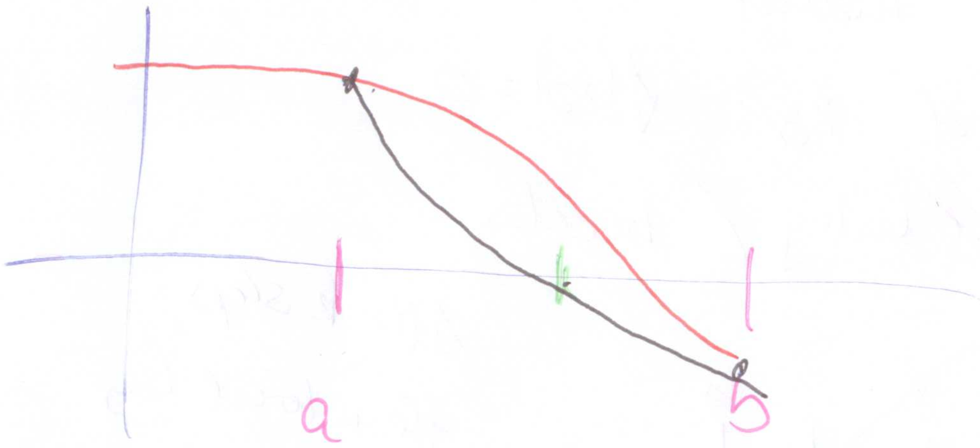


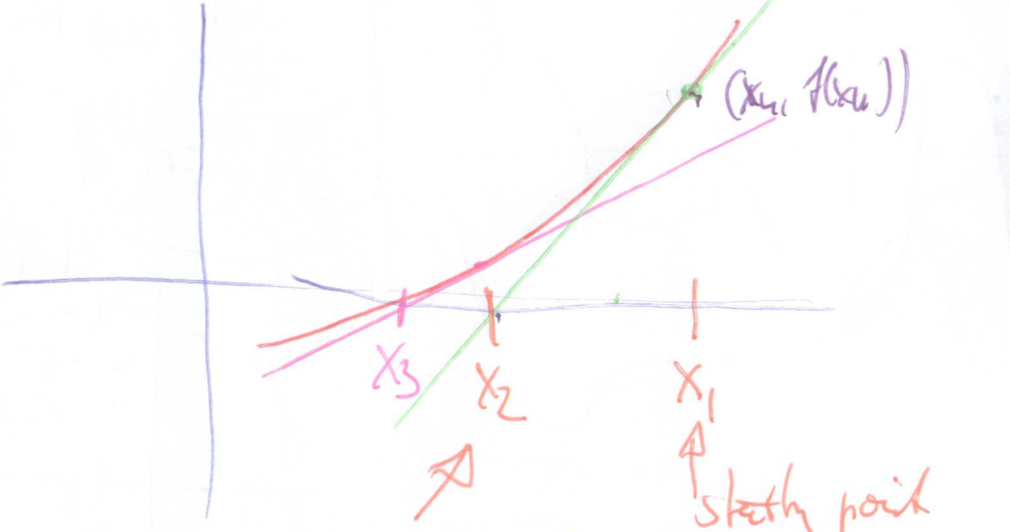
After k steps the interval has length $\frac{1}{2^k} \cdot$ initial

⇝ Initial length 1, if we want solution with error e (say 10^{-6})

We need $-\log_2 \left(\cancel{10} \; e \right)$ steps

Ex: $-\log_2(10^{-6}) \sim 20$





The tangent at x_n goes through the points $(x_n, f(x_n))$ and $(x_{n+1}, 0)$

The next approximation of the place where $f(x_0) = 0$ is the place where the tangent at x_n crosses the x-axis

Slope: $\dfrac{f(x_n) - 0}{x_n - x_{n+1}} = f'(x_n)$ $\xrightarrow{\text{solve for } x_{n+1}}$ $\boxed{x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}}$

Newton's method

Ex: $f(x) = \sin(x)$, $x_0 = 2$

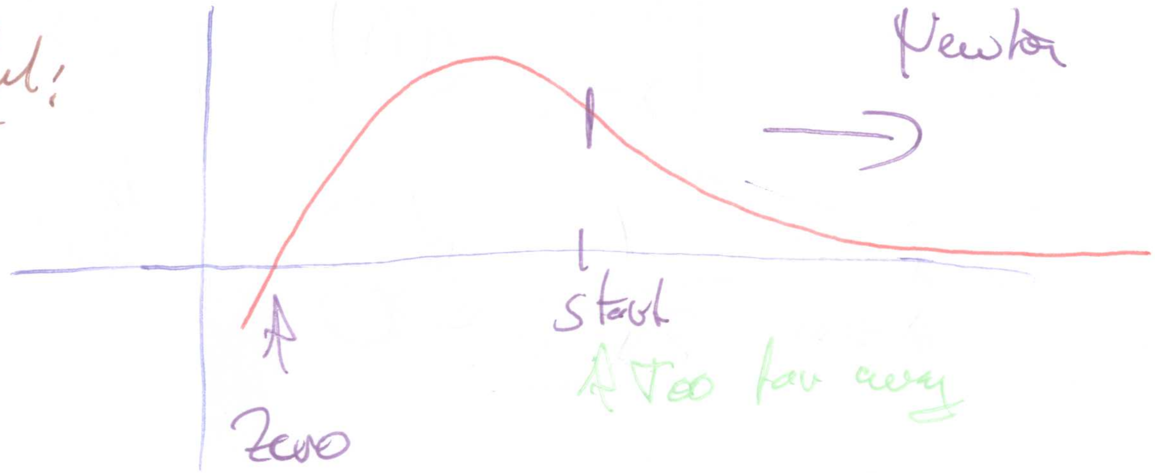
$$x_{n+1} = x_n - \frac{\sin(x_n)}{\cos(x_n)}$$

Iterate

$x_1 = 4.18$, $x_2 = 2.46$, $x_3 = 3.266$, $x_4 = 3.1409$, $x_5 = 3.14159265 4$, Newton's method "converges quadratic"

With each iteration the number of correct digits doubles

Careful!



Growth, order of growth

L'Hospital's rule "lowpytaal"

Find: $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)}$ if both limits exist and not ∞ and not zero

limit rules

Will not work for $\dfrac{0}{0}$ $\dfrac{\pm\infty}{\pm\infty}$

To calculate $\lim\limits_{x\to a} \frac{f(x)}{g(x)}$ if $\lim\limits_{x\to a} f(x) = \pm \lim\limits_{x\to a} g(x) = 0, \pm\infty$

$$= \boxed{\lim_{x\to a} \frac{f'(x)}{g'(x)}}$$

Ex:

$$\lim_{x\to\infty} \frac{x-1}{x^2-1} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\infty} \frac{1}{2x} = 0$$

$f \quad f'$

$2x^3 \rightsquigarrow 6x^2$

$7x^5 \rightsquigarrow 35x^4$

L'Hospital

$$\lim_{x\to\infty} \frac{3x+1}{5x+1} = \lim_{x\to\infty} \frac{3}{5} = \frac{3}{5}$$

derivative

$$f(x) = \sqrt[2]{x} = x^{\frac{1}{2}} \rightsquigarrow \frac{1}{2}\cdot x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$x^{1/3}$ $\quad \sin(x)$

$\frac{1}{3}\cdot x^{-\frac{2}{3}}$ $\quad \cos(x)$

$$f(x) = \sqrt[3]{\sin(x)} = (\sin(x))^{\frac{1}{3}} \overset{\text{deriv.}}{\rightsquigarrow} \frac{1}{3}(\sin(x))^{-\frac{2}{3}}\cdot\cos(x) = \frac{\cos(x)}{3(\sqrt[3]{\sin(x)})^2}$$