Derivative Algorithm :

p. 79/80

Input : Formula for $f(x)$

Output : Formula for $f'(x) = \frac{d}{dx} f(x)$

1) How is f put together ?

2) Case distinction

if $f(x) = g(x) + h(x)$ then $f'(x) = g'(x) + h'(x)$

elif $f(x) = c \cdot g(x)$ then $f'(x) = c \cdot g'(x)$

constant

elif f is elementary then table lookup

elif f is power w. constant exponent: $f(x) = x^n$ then $\boxed{f'(x) = n \cdot x^{n-1}}$

elif: $f(x) = g(x) \cdot h(x)$ then $f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$ ← Product Rule

elif $f(x) = g(h(x))$ then $f'(x) = g'(h(x)) \cdot h'(x)$ ← Chain Rule

elif $f(x) = g(x) / h(x)$ then $f'(x) = \frac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{h^2(x)}$ Quotient Rule

polynomial :

$$f(x) = \sum_{i=0}^{n} a_i x^i$$

$$f'(x) = \sum_{i=1}^{n} i \cdot a_i x^{i-1}$$

Ex : $f(x) = 2x^3 + x + 3$

$f'(x) = 3 \cdot 2 \cdot x^2 + 1$

Includes constants

$f(x) = c \cdot x^0$

$f'(x) = c \cdot \underline{0} \cdot x^{-1} = 0$

elif $f(x) = g(x)^{h(x)}$

$= \exp(\log(g(x)^{h(x)})) = \exp(h(x) \cdot \log(g(x)))$

Use existing rules

fi

end

Note: $\frac{d}{d\boxed{x}} f(x)$ derivative wrt to particular variable x.

Thus $\frac{d}{dx} y = 0$

not x, different

$f^{(1)}(x) = f'(x)$ is function. So can calculate the derivative:

$f''(x) = f^{(2)}(x)$ — " —

$f'''(x) = f^{(3)}(x)$

Ex. Why $lg'(x) = \frac{1}{x}$?

$x = lg(\exp(x))$ | derivative

$1 = lg'(\cancel{\exp(x)}) \cdot \cancel{\exp'}(x)$ | change variable $y = \exp(x)$

$\exp(x) \to y$, $\exp' \to \exp \to y$

$1 = lg'(y) \cdot y$ | $\cdot \frac{1}{y}$

$= \frac{1}{\cancel{y}} = lg'(\cancel{y})$, $y \to x$

$f(x)$

$g(x) \circ$ composition

$g'(x)$

$h(x)$, x^{-1}

$h'(x)$, $-\frac{1}{x^2}$

$-\frac{1}{h^2(x)} \cdot h'(x)$

Derivative of Quotient:

$$f(x) = \frac{g(x)}{h(x)} = g(x) \cdot (h(x))^{-1}$$

$$= \frac{g'(x)}{h(x)} \quad \left[= g'(x) \cdot \cancel{(h(x))^{-1}}\right] + g(x) \cdot \left(-\frac{h'(x)}{h^2(x)}\right)$$

$$= \frac{g'(x)\,h(x) - g(x) \cdot h'(x)}{h^2(x)}$$ Quotient Rule

Ej: $f(x) = \frac{x^2}{\sin(x)}$

$$f'(x) = \frac{2x \cdot \sin(x) - x^2\cos(x)}{\sin^2(x)}$$

$$f''(x) = \frac{(2+x^2)\cdot\sin(x)\cdot\sin^2(x) - \left(2x\sin(x) - x^2\cos(x)\right)2\sin(x)\cos(x)}{\sin^4(x)}$$