Derivative Algorithm : (p. 79/80)

Input : Formula for $f(x)$

Output : Formula for $f'(x) = \frac{d}{dx} f(x)$

1) How is f put together ?

2) Case distinction

if $f(x) = g(x) + h(x)$ then $f'(x) = g'(x) + h'(x)$

elif $f(x) = c \cdot g(x)$ then $f'(x) = c \cdot g'(x)$
(c: constant)

elif f is elementary then ← the lookup
includes constants
$f(x) = c \cdot x^0$
$f'(x) = c \cdot 0 \cdot x^{-1} = 0$

elif f is power w. constant exponent: then $f(x) = x^n$ → $f'(x) = n \cdot x^{n-1}$

elif: $f(x) = g(x) \cdot h(x)$ then $f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$ ← Product Rule

elif $f(x) = g(h(x))$ then $f'(x) = g'(h(x)) \cdot h'(x)$ ← Chain Rule

polynomial :

$$f(x) = \sum_{i=0}^{n} a_i x^i$$

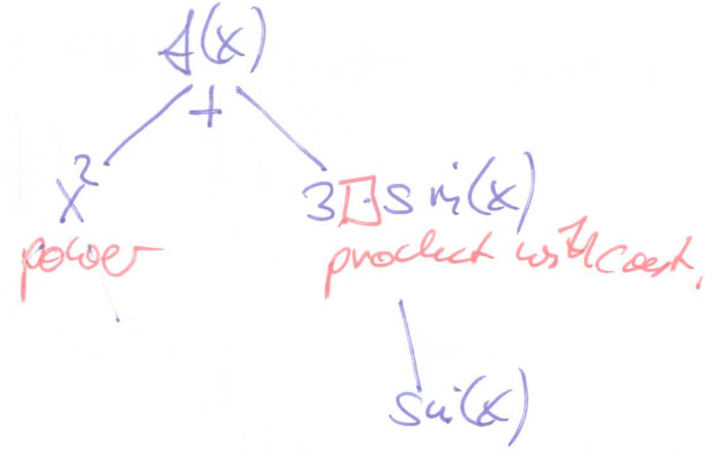
$$f'(x) = \sum_{i=1}^{n} i \cdot a_i x^{i-1}$$

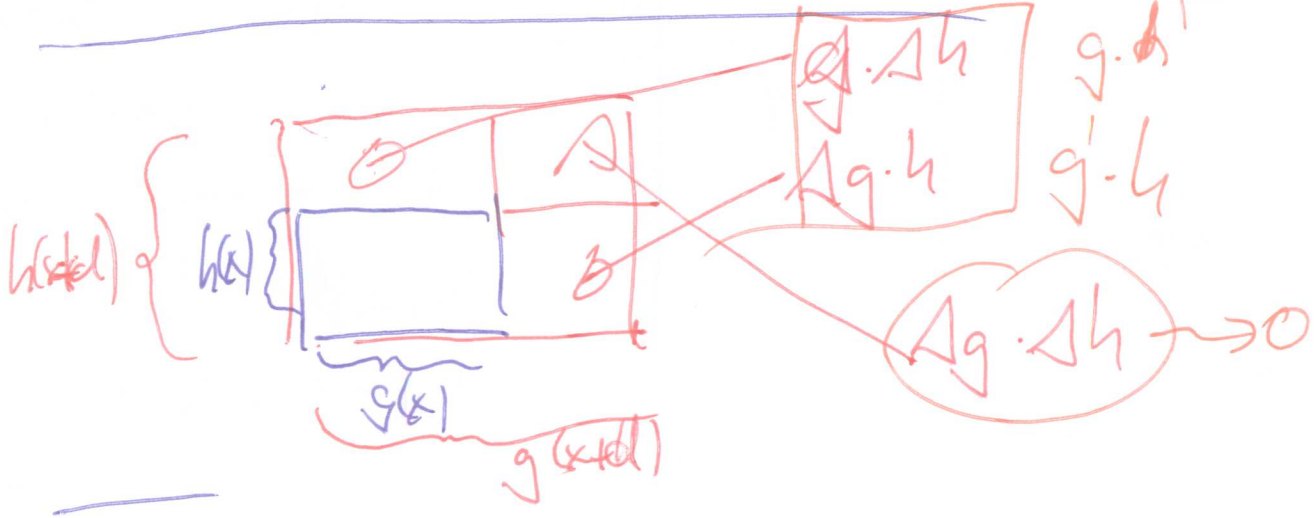
ex : $f(x) = 2x^3 + x + 3$

$f'(x) = 3 \cdot 2 \cdot x^{2} + 1$

Ex: $f(x) = x^2 + 3 \cdot \sin(x)$

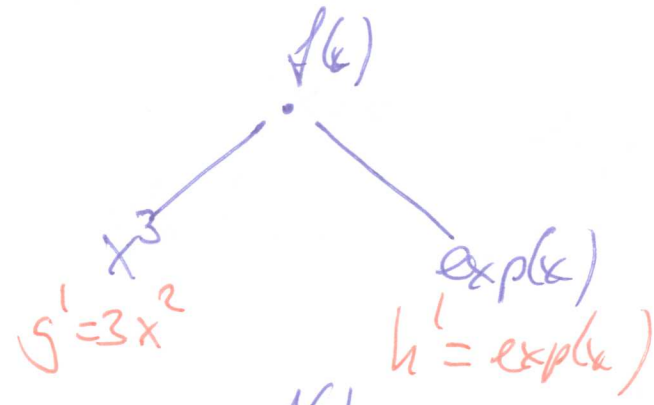
$f'(x) = 2x + 3 \cdot \cos(x)$





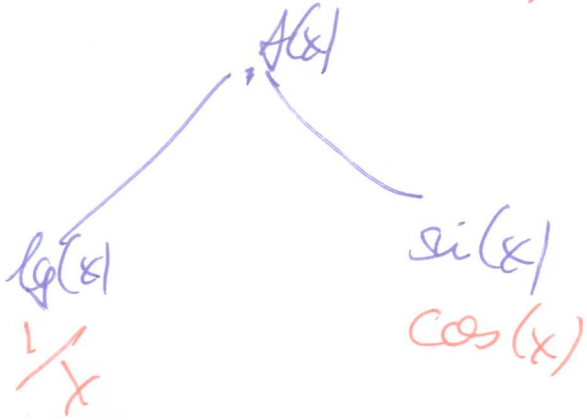
Ex: $f(x) = x^3 \cdot \exp(x)$

$f'(x) = 3x^2 \cdot \exp(x) + x^3 \cdot \exp(x)$



$f(x) = \log(x) \cdot \sin(x)$

$f'(x) = \frac{1}{x} \sin(x) + \log(x) \cdot \cos(x)$



Chain Rule: $f(x) = g(h(x))$

$$\frac{df}{dx} = \frac{df}{dh} \cdot \frac{dh}{dx}$$

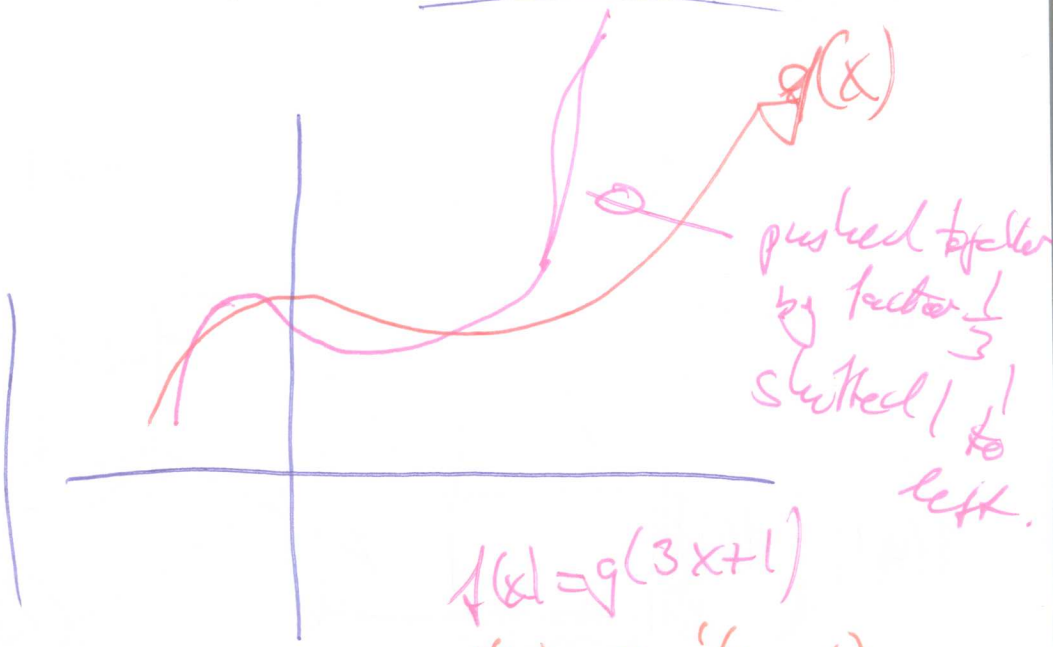
$f'(x) = g'(h(x)) \cdot h'(x)$

Ex: $f(x) = (\cos(x))^2$

$f'(x) = 2(\cos(x)) \cdot (-\sin(x))$

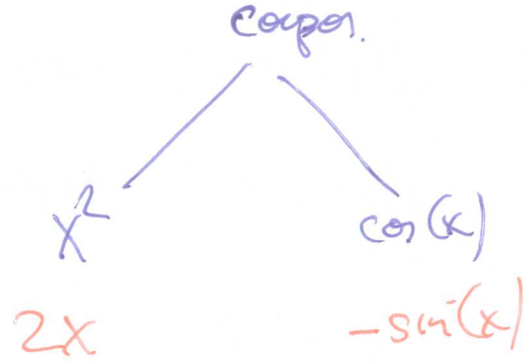
$f(x) = \cos(x^2)$

$f'(x) = -\sin(x^2) \cdot 2x$



$f(x) = g(3x+1)$

$f'(x) = 3 \cdot g'(3x+1)$



3 months:

$$S \cdot 1.003^3 - 1000 - 1000 \cdot 1.003 - 1000 \cdot 1.003^2$$

k months:

$$S \cdot 1003^k - \left(\sum_{i=0}^{k-1} \underbrace{1000}_{a} \cdot \underbrace{1.003^i}_{r} \right)$$

$$\sum_{i=0}^{n} a \cdot r^i = a \cdot \frac{1-r^{i+1}}{1-r}$$

$$= 1000 \cdot \frac{1-1.003^k}{1-1.003}$$

$$A = S \cdot 1.003^k - 1000 \cdot \frac{1-1.003^k}{1-1.003}$$

b): Set $k=360$, want 0

$$0 = S \cdot 1.003^{360} - 1000 \cdot \frac{1-1.003^k}{1-1.003} \Rightarrow S = \frac{1000 \cdot \frac{1-1.003^{360}}{1-1.003}}{1.003^{360}}$$

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c):

$$0 = S \cdot \underbrace{1003^k}_{x} - 1100 \cdot \frac{1 - \underbrace{1.003^k}_{x}}{1-1.003}$$

$$= S \cdot x - \frac{1100}{1-1.003} + \frac{1100}{1-1.003} \cdot x = \frac{1100}{1-1.003} \cdot \left(S + \frac{1100}{1-1.003} \right) x$$

$$\Rightarrow \frac{1100}{1-1.003} = x\cdot\left(5+\frac{1100}{1-1.003}\right) \Rightarrow x = \frac{1}{\left(5+\frac{1100}{1-1.003}\right)}\cdot\frac{1100}{1-1.003}$$

$$\text{and} \quad x = 1.003^{k} \quad \Rightarrow \quad k = \log_{1.003}(x) = \frac{\log(x)}{\log(1.003)}$$