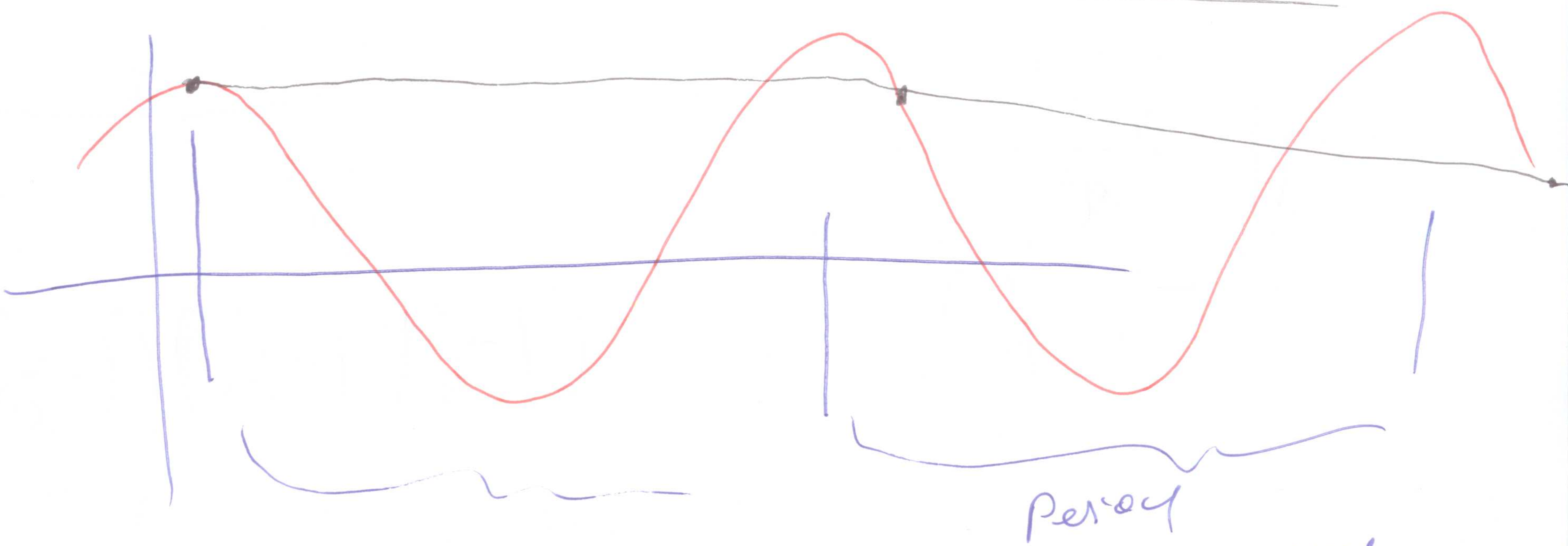
~~b_i~~ a ~~a_i~~ b

$$a_i = b_i - b_{i-1}$$

b: totals
a: daily

$$b_i = \sum_{n=1}^{i} a_n + \text{initial balance}$$

Aliasing



Need to sample at least twice per period

Shannon-Nyquist theorem "Nyquist Frequency" over a longer range:

b_i 0, 8, 15, 21, 24, 26, 28, 26, 25, 23, 21, 19, 18, 16, 15, 14, 13, 14, 15, 16, 18, 20, 22, 25, 28, 31, 34, 37, 40, 43, 46, 48, 50, 51, 52, 53, 52, 51, 49, 48, 45, 42, 40, 36, 33, 31, 28, 27, 26, 27, 30

We get a sequence of changes $\{a_i\}$ (starting with a_1) as $a_i = b_i - b_{i-1}$:

8, 7, 6, 3, 2, 2, −2, −1, −2, −2, −2, −1, −2, −1, −1, −1, 1, 1, 1, 2 ← derived sequence

The values of both sequences are depicted (with points connected) in Figure V.3.

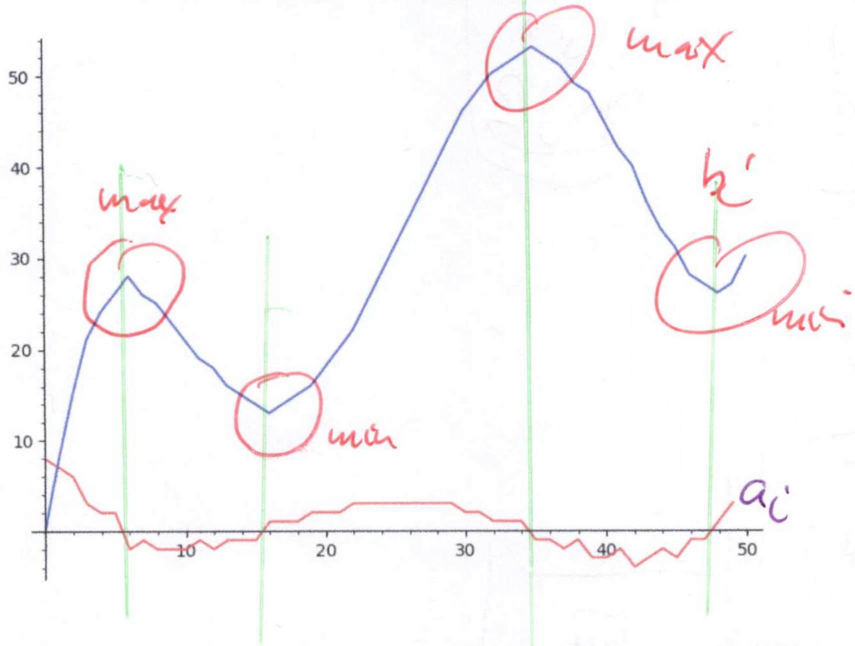


Figure V.3: A sequence and its derivative

An obvious question one can ask for such a sequence b_i is for what the maximum and minimum (largest and smallest) values over the investigated period are. (In the previous example these would have been the lowest and the highest worth of the business.) We mark the areas where the function is (locally, that is in relation to its neighbors) maximal or minimal in Figure V.4.

We note that at these index values (for which the function bn is maximal, respectively minimal), which are aligned along the x-axis, the derivative is zero. (An eagle-eyed reader might notice that we are slightly cheating here: Since we sum up the values of the derivative, the maximum happens at the x-value plus 1, and our derivatives are only close to zero. This will be resolved later when we will decrease the step-width more and more.)

Furthermore, at an index i, where b_i has minimum value, the derivative a_i changes from being negative to being positive. And at the index i where b_i has maximum value, the derivative changes sign from positive to negative. The reason

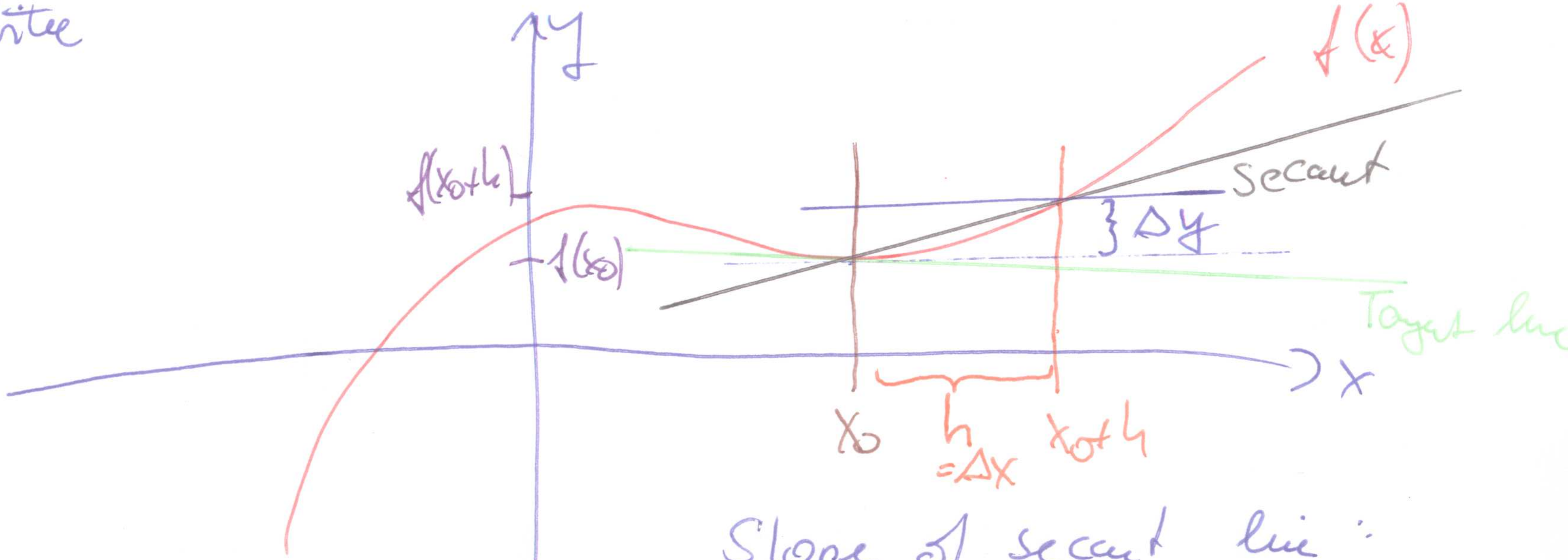
$$(a_0 + a_1 x + a_2 x^2) \cdot (b_0 + b_1 x + b_2 x^2)$$

$$= \boxed{a_0 b_0} + \boxed{(a_1 b_0 + a_0 b_1)} \cdot x + \boxed{(a_0 b_2 + a_1 b_1 + a_2 b_0)} \cdot x^2 + \ldots$$

$$+ \ldots + \left(\sum_{j=0}^{?} a_j \cdot b_?\right) x^i + (\ldots) x^{?}$$

$$\sum_{j=0}^{3} a_j \cdot b_{\boxed{?}}$$

Derivative



Slope of secant line:

$$\frac{\Delta y}{\Delta x} = \frac{f(x_0+h) - f(x_0)}{\cancel{x_0}+h \cancel{- x_0}}$$