$$\sum_{i=0}^{n} a \cdot r^i = a + a \cdot r + a \cdot r^2 + \cdots + a r^n = a \frac{1 - r^{n+1}}{1 - r}$$

Thus $\lim_{n \to \infty} = \sum_{i=0}^{\infty} a \cdot r^i = a \cdot \frac{1}{1-r}$ if $|r| < 1$

Ex: Express $0.\overline{08} = 0.\underbrace{08}\underbrace{08}\underbrace{08}08\ldots$ as fraction

$a = \frac{8}{100}$ $\quad \frac{1}{100} \cdot 8 + \frac{1}{100^2} 8 + \frac{1}{100^3} \cdot 8 \ldots = \sum_{i=0}^{\infty} \frac{8}{100} \left(\frac{1}{100}\right)^i$

$r = \frac{1}{100}$

$$= \frac{8}{100} \frac{1}{1 - \frac{1}{100}} \qquad 1 - \frac{1}{100} = \frac{99}{100} \qquad = \frac{100}{99}$$

$$= \frac{8}{99}$$

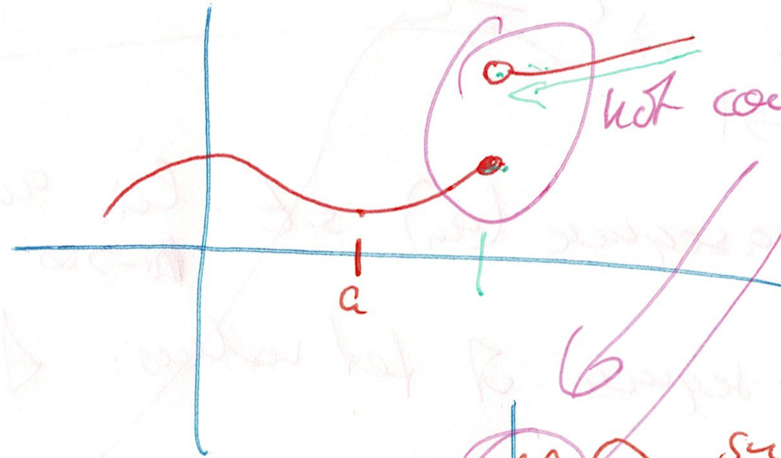
$$\underbrace{a + \cdots + a}_{n} = n \cdot a$$

$$1 + 2 + 3 + 4 + \cdots + n = \frac{n(n+1)}{2}$$

$$\lim_{x\to a} f(x) = \lim_{n\to\infty} f(a_n)$$

if $\lim_{n\to\infty} a_n = a$

and limit the same regardless of sequence.

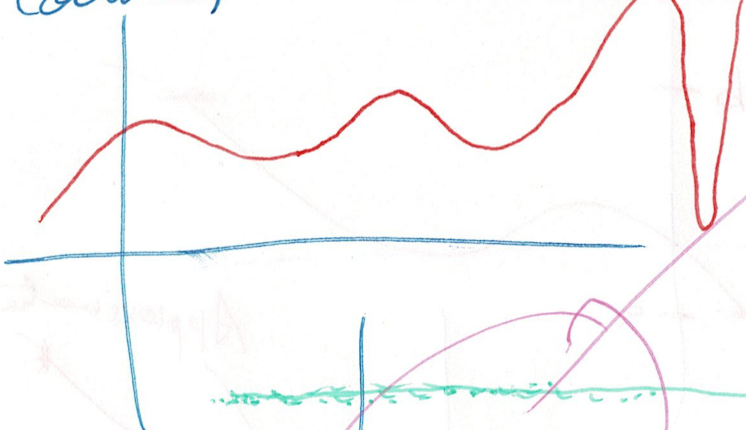


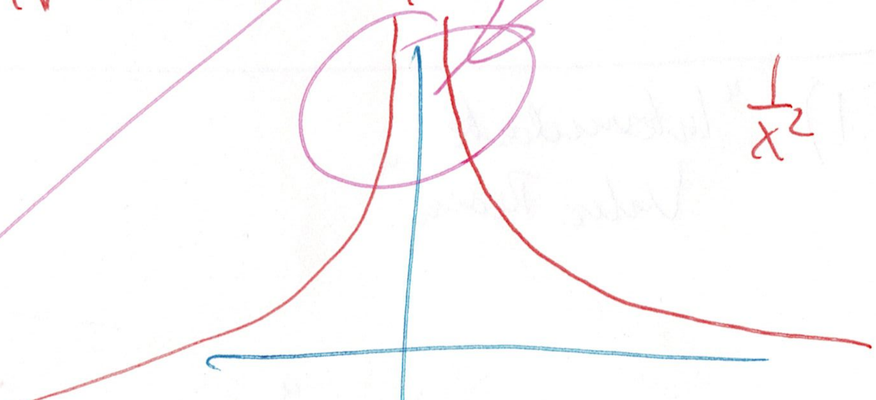
$\sin\left(\frac{1}{x}\right)$

"Typically": $\lim_{x\to a} f(x) = f(a)$

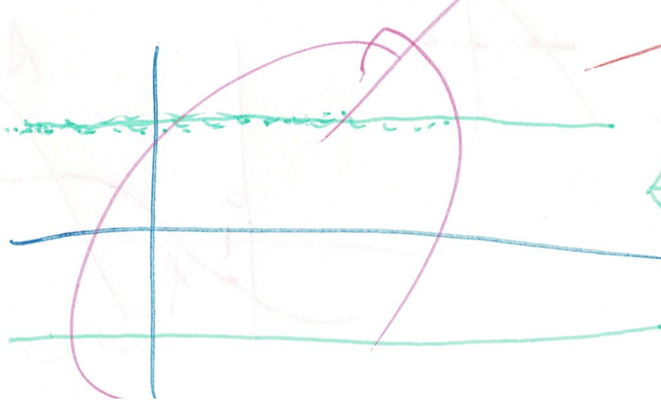
If this is the case, f is continuous at a.

Continuous





$\frac{1}{x^2}$



$$f(x) = \begin{cases} 1 & x \in \mathbb{Q} \\ -1 & x \notin \mathbb{Q} \end{cases}$$

Ex: $\lim_{x \to 1} x^2 = ?$

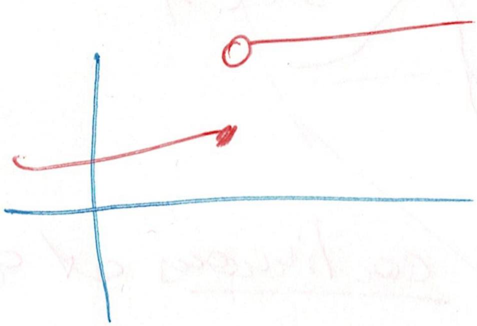
My choice: $a_n = 1 + \frac{1}{n}$

Take a sequence (a_n) s.t. $\lim_{n \to \infty} a_n = 1$

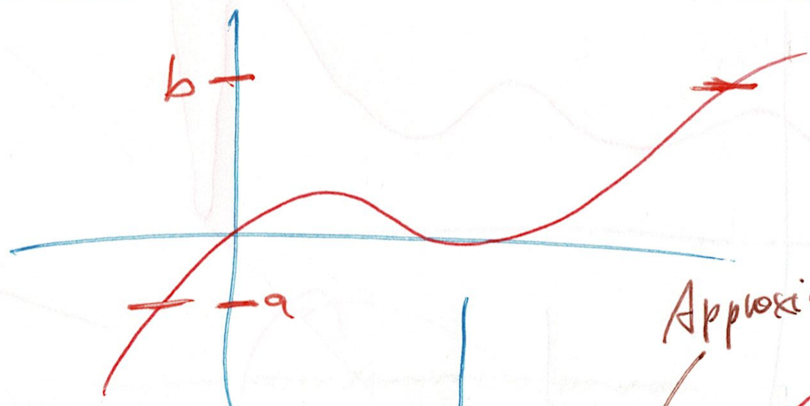
Now sequence of fct values: $f_n = \left(1 + \frac{1}{n}\right)^2 = \boxed{1 + \frac{2}{n} + \frac{1}{n^2}}$

$\lim_{n \to \infty}$

1

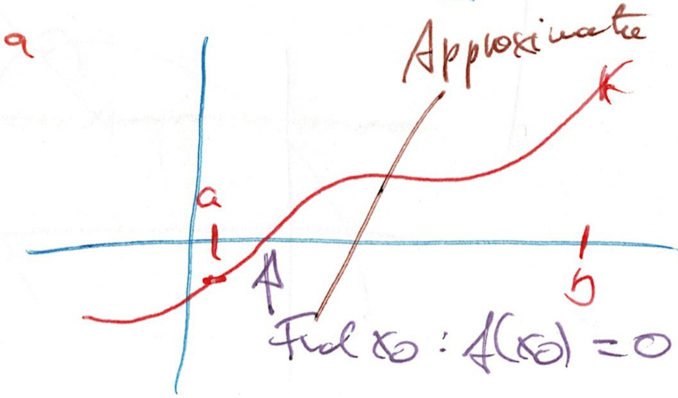


Uses: 1) "Intermediate Value Theorem".



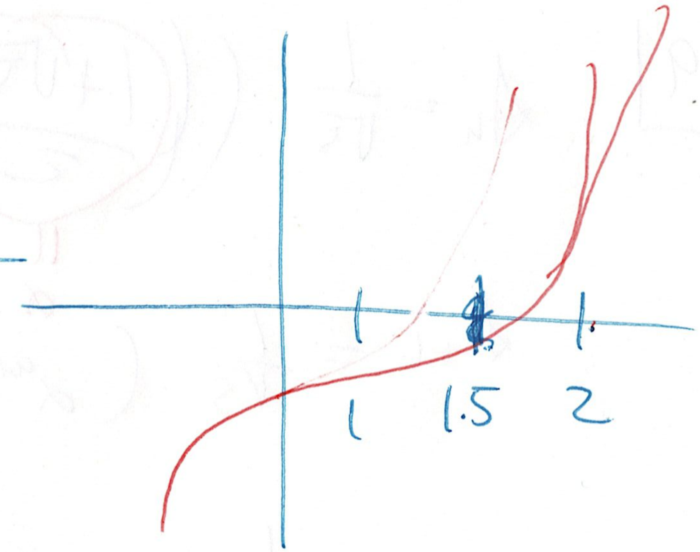
2) "Interval Halving", "Bisection method"

$f(a) < 0$, $f(b) > 0$



ex: $f(x) = x^3 - 7$

a	1	1.5	
b	2	2	
f(a)	-6	-3.6	
f(b)	1	1	
$\frac{a+b}{2}$ Middle	1.5	1.75	
f(Middle)	-3.6..	-1.64	



How good? After n steps, interval has length $\frac{\ell}{2^n}$ = initial length

Suppose initial length is 1. Want result to k digits after the decimal point : $\underbrace{|b-a|}_{\frac{1}{2^4} = 2^{-4}} \le 10^{-k}$ $\Longrightarrow$ $n \ge \log_2(10^k)$

Apply $\log_2(x)$ change sign

$= k \cdot \underbrace{\log_2(10)}_{2.\dots}$

3 to 4 steps for each digit

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$$f_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$$

$\frac{1+\sqrt{5}}{2} = \alpha$, $\frac{1-\sqrt{5}}{2} = \beta$

$$f_{n+1} = \frac{1}{\sqrt{5}}\cdot\left(\alpha^{n+1} - \beta^{n+1}\right), \quad f_{n+2} = \frac{1}{\sqrt{5}}\cdot\left(\alpha^{n+2} - \beta^{n+2}\right)$$

$$f_n + f_{n+1} = \frac{1}{\sqrt{5}}\left(\alpha^n - \beta^n\right) + \frac{1}{\sqrt{5}}\left(\alpha^{n+1} - \beta^{n+1}\right) = \frac{1}{\sqrt{5}}\left(\alpha^n + \alpha^{n+1} - \left(\beta^n + \beta^{n+1}\right)\right)$$

$$= \frac{1}{\sqrt{5}}\left(\alpha^n \underbrace{(1+\alpha)}_{=\alpha^2} - \beta^n \underbrace{(1+\beta)}_{=\beta^2}\right)$$

$$\alpha^n + \alpha^{n+1} = \alpha^n\cdot(1+\alpha)$$

$$\beta^n + \beta^{n+1} = \beta^n(1+\beta)$$

$$= \frac{1}{\sqrt{5}}\left(\underbrace{\alpha^{n+2}}_{\alpha^n\cdot\alpha^2} - \underbrace{\beta^{n+2}}_{\beta^n\cdot\beta^2}\right)$$

$$= f_{n+2}$$

$$\alpha^2 = \frac{(1+\sqrt{5})^2}{4} = \frac{1+2\sqrt{5}+5}{4} = \frac{3\cancel{6}+\cancel{2}\sqrt{5}}{\cancel{4}2}$$

$$1+\alpha = \frac{1+\sqrt{5}}{2}+1 = \frac{3+\sqrt{5}}{2}$$

$$1+\beta = \frac{3-\sqrt{5}}{2}$$