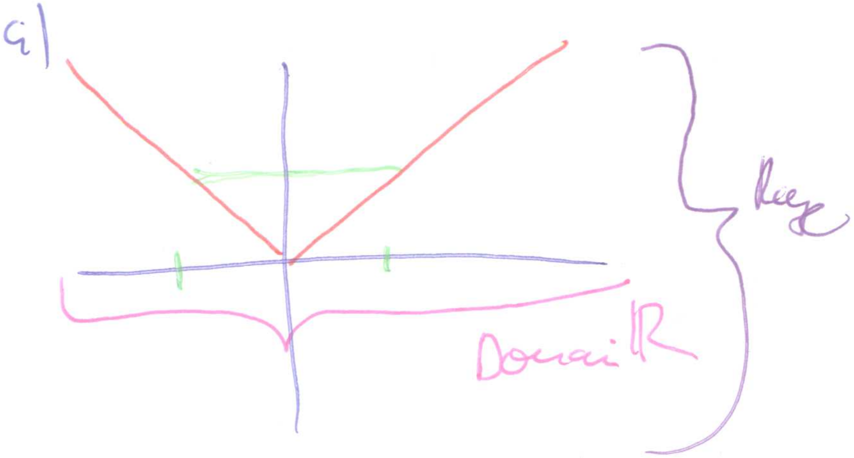
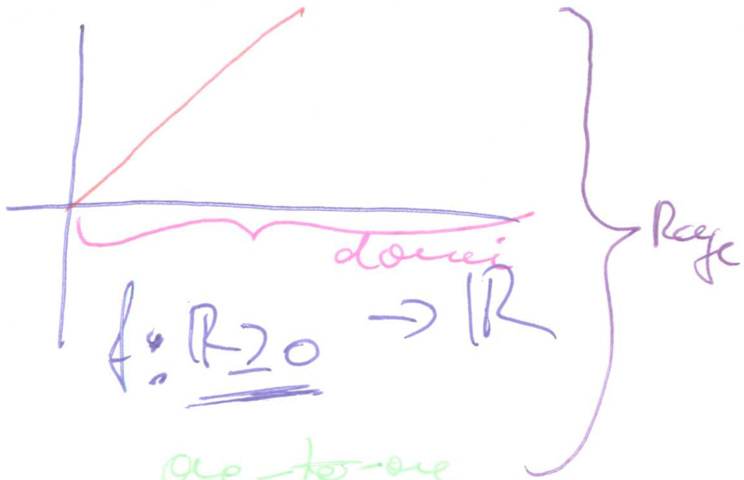
a)

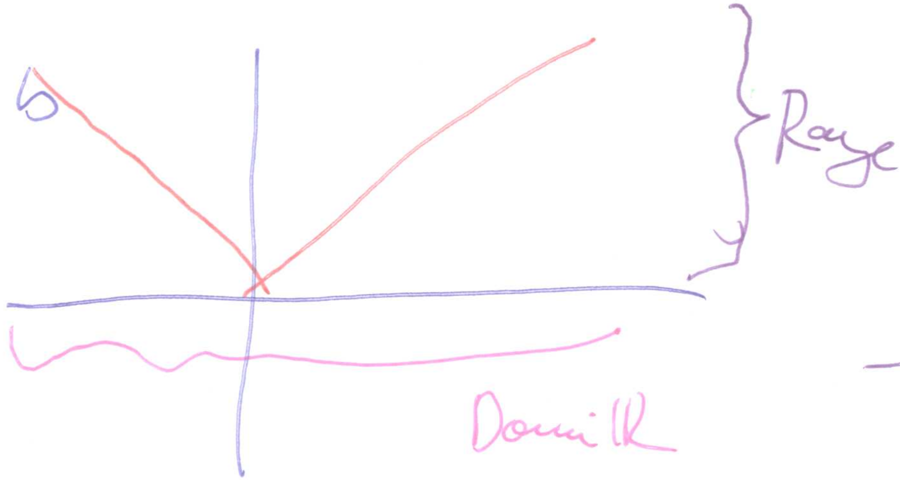




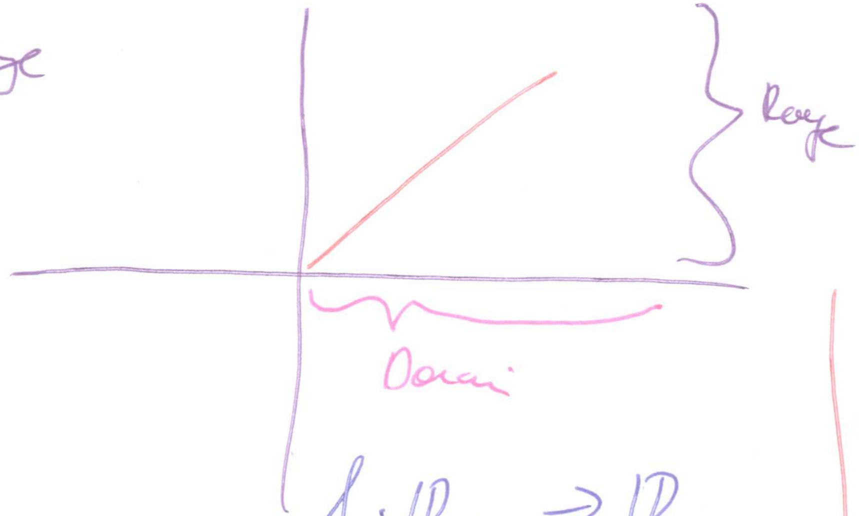
$f: \mathbb{R}_{\geq 0} \to \mathbb{R}$

one-to-one
not onto

b)



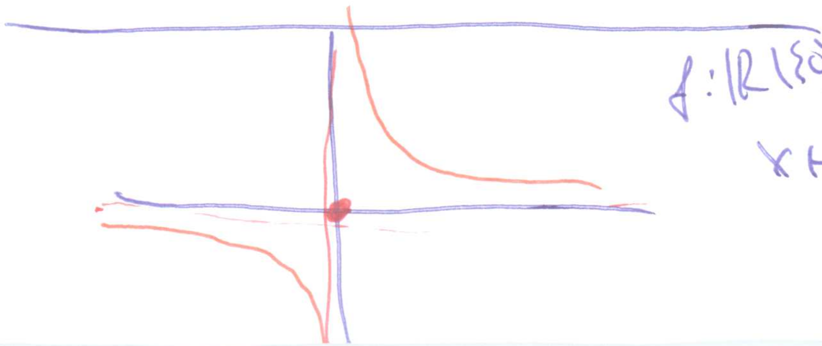
not one-to-one
onto



$f: \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$

$x \mapsto |x|$

onto + one-to-one



$f: \mathbb{R}\setminus\{0\} \to \mathbb{R}\setminus\{0\}$

$x \mapsto \frac{1}{x}$

$x \mapsto \begin{cases} \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

Sequence: $a_i = 3 + \frac{1}{i}$

$a_1 = 4,\ a_2 = 3 + \frac{1}{2} = 3\frac{1}{2},\ a_3 = 3 + \frac{1}{3} = 3\frac{1}{3},\ a_4 = 3\frac{1}{4}, \ldots$

Properties of this sequence:

- decreasing $\quad a_{i+1} \lneqq a_i$

strictly

- bounded from above: $a_i \leq 4$ for all i

below: $a_i \geq 3$

monotonous

$b_i = 3 + (-1)^i$

$b_1 = 2,\ b_2 = 4,\ b_3 = 2, \quad 4, 2, 4, 2, 4, \ldots$

bounded, not increasing
not decreasing

$c_i = i^2$: $c_1 = 1,\ c_2 = 4,\ c_3 = 9, \ldots$

increasing
not bounded from above

Show: ~~f~~ $x \to |x|$ is onto $\mathbb{R}_{\geq 0}$ as:

Let $a \in \mathbb{R}_{\geq 0}$. [Task: Find x, s.t. $|x| = a$]

solve for x

Let $x = a$. Then $|x| = |a| = a$.

Show: $x \mapsto x^2 - 2x + 1$ is onto $\mathbb{R}_{\geq 0}$

Let $a \geq \mathbb{R}_{\geq 0}$ [Solve: $\underbrace{x^2 - 2x + 1}_{=(x-1)^2} = a$

$\Rightarrow x = \sqrt{a} + 1$]

Root exists as $a \geq 0$

Then sol: $x = \sqrt{a} + 1$.

Then $f(x) = x^2 - 2x + 1$

$= (\sqrt{a}+1)^2 - 2(\sqrt{a}+1) + 1$

$= \sqrt{a}^2 \cancel{+ 2\sqrt{a}} \cancel{+1} \cancel{- 2\sqrt{a}} \cancel{- 2} \cancel{+1} = \sqrt{a}^2 = a.$

For one-to-one:

Show: If

$x^2 - 2x + 2 =$

$y^2 - 2y + 2$ then

$x = y$

(Note: false)

Convergence: A sequence (a_i) is convergent to a limit L

"If the values of the sequence get closer and closer to L"

write $\lim_{i \to \infty} a_i = L$

Ex: $a_i = 4$ convergent, $\lim_{i \to \infty} a_i = 4$

$b_i = 4 + \frac{1}{i}$ convergent, $\lim_{i \to \infty} b_i = 4$

$5, 4\frac{1}{2}, 4\frac{1}{3}, 4\frac{1}{4}, 4\frac{1}{5}, \ldots, 4\frac{1}{1000000}, \ldots$

$c_i = 4 + \frac{(-1)^i}{i}$ convergent, $\lim_{i \to \infty} c_i = 4$

$3, 4\frac{1}{2}, 3\frac{2}{3}, 4\frac{1}{4}, 3\frac{4}{5}, \ldots$

$d_i = i^2$ not convergent (Maybe: $\lim_{i \to \infty} d_i = \infty$)

$e_i = 4 + (-1)^i$ not convergent

3, 5, 3, 5, 3, 5 ......

$f_i = \begin{cases} 4 + 1/i & \text{if } i \text{ even} \\ 5 & \text{if } i \text{ odd} \end{cases}$ not convergent

$5, 4\tfrac{1}{2}, 5, 4\tfrac{1}{4}, 5, 4\tfrac{1}{6}, 5, 4\tfrac{1}{8}, 5, \ldots$

$g_i = \begin{cases} 4 + 1/i & i \text{ even} \\ 4 & i \text{ odd} \end{cases}$

---

ex $h_i = \dfrac{6i+3}{5i+2}$

plug in values of i

$i = 10$: 1.211

$i = 100$: 1.2011

$i = 10^6$: 1.2000000120

$i = 10^{10}$: 1.200000

$\lim h_i = 1.2 = \frac{6}{5}$