$$f_{n+1} = \frac{1}{\sqrt5}\left(\left(\frac{1+\sqrt5}{2}\right)^{n+1} - \left(\frac{1-\sqrt5}{2}\right)^{n+1}\right)$$

$n+2$ $\quad$ $n+2$ $\quad$ $n+2$

Formula

Start.

$f_n + f_{n+1}$ $\quad$ show f_{n+2}

α $\quad$ β $\quad$ γ

"2"

$a_1 = 2,\quad a_{n+1} = 2a_n + 5$

$a_{n+1} = 2,\ n=1$

$a_2 = 2\cdot a_1 + 5 = 9$

$a_3 = 2\cdot a_2 + 5 = 23$

a_4

[9 6)]: Suppose $f_n = 3(n+1) + \sqrt2\cdot n$

$f_{n+1} = 3(n+2) + \sqrt2(n+1)$

$f_{n+2} = 3(n+3) + \sqrt2(n+2)$

WRONG FORMULA

$f_n + f_{n+1} = 3(n+1) + \sqrt2(n) + 3(n+2) + \sqrt2(n+1)$

$= 3(n+1+n+2) + \sqrt2(n+n+1)$

$= \ldots$

f_{n+2}

Stop!

Museum Visitors grows by 5% each year.

$r=1.05$

Y_0 $\quad$ Y_1

$a_0=100 \quad a_1=a_0\cdot r=105, \quad a_2=a_1\cdot r=a_0\cdot r^2=\ldots$

Q: How many people have been in total after k years

Calculate $P_k = a + \cancel{a\cdot r} + \cancel{ar^2} + \cdots + \cancel{a\cdot r^k} = \sum_{i=0}^{k} a\cdot r^i$

To get formula calculate $r\cdot P_k = \cancel{r\cdot a} + \cancel{a\cdot r^2} + ar^3 + \cdots \cancel{\ } + ar^{k+1}$

Then $\underbrace{P_k - r\cdot P_k}_{P_k\cdot(1-r)} = a - a\cdot r^{k+1} = a\cdot(1-r^{k+1})$

$\Rightarrow \boxed{P_k = a\dfrac{1-r^{k+1}}{1-r}}$

Ex: Museum: After 10 years: $100\cdot\dfrac{1-1.05^{11}}{1-1.05} \approx 1420.67\ldots$

50 $\quad$ 51

Ex: 64 spaces: 1, 2, 4, 8, ... rice corns. How many in total.

$a=1 \quad r=2$

$P_{63} = 1\cdot\dfrac{(1-2^{64})}{1-2} = 2^{64}-1 \approx 1.84\cdot 10^{19}$

Ex: $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$

$a = 1$ $\quad r = \frac{1}{2}$ $\quad$ Sum $P_3 = 1 \cdot \frac{1 - \left(\frac{1}{2}\right)^4}{1 - \frac{1}{2}}$

After k: $1 \cdot \frac{1 - \left(\frac{1}{2}\right)^{k+1}}{1 - \frac{1}{2}}$

If $|r| < 1$ then the sequence P_k converges to

$$\sum_{i=0}^{\infty} a \cdot r^i = a \cdot \frac{1}{1-r}$$

In example: $1 \cdot \frac{1}{1 - \frac{1}{2}} = 2$

Ex (Mortgage) Loan: L dollars.

Every month pay back C dollars.

Interest: each month factor r

Interest: 6%/year $= \frac{1}{2}$% per month $\Rightarrow r = 1.005$

Ex: $L = 200000$,

b_k = Outstanding loan after k ~~periods~~ months

$b_0 = 200000$. $\quad b_1 = 201000 - c$

$b_1 = L \cdot r - c$

$b_2 = b_1 \cdot r - c$

$= L \cdot r^2 - c \cdot r - c$

$= L \cdot r^2 - c \cdot (1 + r)$

After k months:

$$b_k = L \cdot r^k - \overbrace{(C + Cr + Cr^2 \cdots + Cr^{k-1})}^{C \cdot \sum_{i=0}^{k-1} r^i} =$$

Outstanding Loan sum after k months is

$$L \cdot r^k - \sum_{i=0}^{k-1} c \cdot r^i = \boxed{L \cdot r^k - c \frac{1-r^k}{1-r}}$$

Mortgage Value after k months.

Ex: $L = 200000$, $r = 1.005$

Solve for c

Pay off after 30 years $k = 30 \cdot 12 = 360$

$$c = L \cdot r^k \frac{1-r}{1-r^k} = 1199.10\ldots, \quad \Big| \; b_{360} = 0$$