$1, 3, 5, 9, -2, \dots$

Let $\mathbb{N}_0 = \{0, 1, 2, 3, \dots\}$

A *sequence* is a function $a: \mathbb{N}_0 \to \mathbb{R}$

$a(0) = 1$, $a(1) = 3$, $a(2) = 5$, ...

a_0 $\quad$ a_1 $\quad$ a_2

Write $(a_i)_i$ for whole sequence
while a_i is the value at i.

$b_i = i^2 + 5$ $\quad$ $b_0 = 5$, $b_1 = 6$, $b_2 = 9$, ...

$c_i = \sum_{j=0}^{i} j$ (Sum of numbers up to i), $c_0 = 0$, $c_1 = 1$, $c_2 = 3$, ...

Note: "Next value" questions are bad

Ex: 2, 4, 6, 8, 10 (Starting at $i=1$)

Guess: $a_i = 2i$
Next value $= 12$

$a_i = \frac{-29}{120} x^5 + \frac{29}{8} x^4 - \frac{493}{24} x^3 + \frac{435}{8} x^2 - \frac{3853}{60} x + 29$ $\quad$ Next: -17

Recursion:

- give (some) initial values.
- Formula how next is obtained from previous.

$a_1 = 3$, $a_i = a_{i-1} + 2$

initial — recursive formula.

explicit form.

Claim:

$a_i = 1 + 2 \cdot i$

Evaluate: $a_2 = a_{2-1} + 2 = 3 + 2 = 5$

$a_3 = a_2 + 2 = 7, \ldots$

How do we verify?

→ verify the initial value: $a_1 = 1 + 2 \cdot 1 = 3$

+ verify the recursive formula for ~~arbitrary~~ index variable

$a_{i-1} = 1 + 2 \cdot (i-1) + 2 = 1 + 2 \cdot i \cancel{-2} \cancel{+2} = 1 + 2i$

Ex "Sum of numbers up to i"

$S_2 = 1+2 = 3$, $S_3 = 1+2+3 = 6$, $S_4 = 1+2+3+4 = 10$

Recursion $\begin{cases} S_1 = 1 \\ S_{i+1} = S_i + (i+1) \end{cases}$

$S_i = S_{i-1} + i$

Claim: $S_i = \dfrac{i(i+1)}{2}$

Verify this formula.

I.V.: $S_1 = \dfrac{1(1+1)}{2} = 1$ ✓

Recursion: $\Rightarrow S_i + (i+1) = \dfrac{i(i+1)}{2} + (i+1) = \dfrac{i^2+i+2i+2}{2}$

$= \dfrac{i^2+i+2i+2}{2} = \dfrac{(i+1)(\overset{(i+2)}{\cancel{i+1+1}})}{2} = S_{i+1}$ ✓

Ex: Fibonacci Numbers

$f_0 = 0$, $f_1 = 1$, $f_{n+2} = f_{n+1} + f_n$

$f_2 = 0+1 = 1$, $f_3 = f_1 + f_2 = 1+1 = 2$, $f_4 = f_3 + f_2 = 1+2 = 3$,

5, 8, 13, 21, ...

Def: Let (a_i) be a sequence. This sequence is

a) increasing if $a_{i+1} \geq a_i$

b) strictly increasing if $a_{i+1} \underset{\neq}{>} a_i$

In Ex: a_i is increasing, as $a_{i+1} = 5 - \frac{1}{i+1} > 5 - \frac{1}{i} = a_i$

Ex:

$a_i = 5 - \frac{1}{i}$

$b_i = 5 + i$

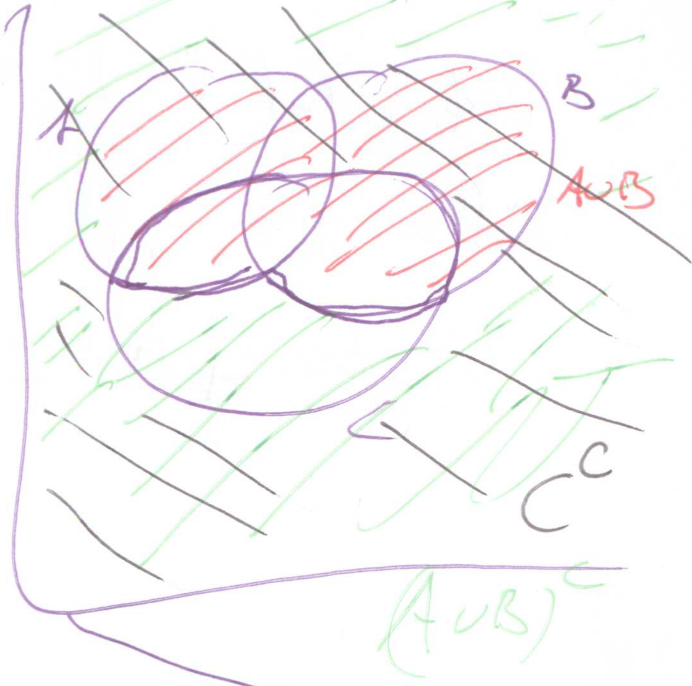
$c_i = 5 + (-1)^i$

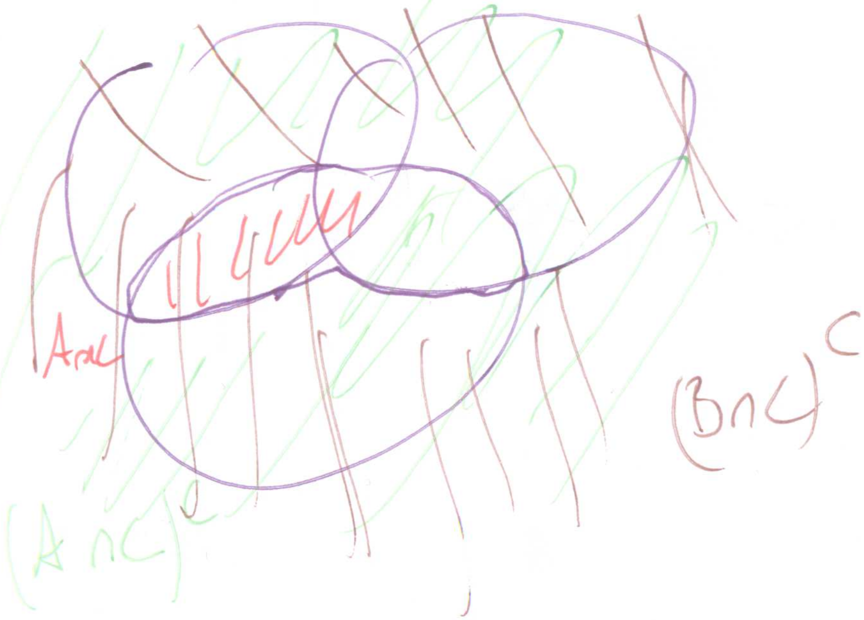
$d_i = 5$

$$(A\cup B)^c \cup C^c \stackrel{!}{=} (A\cap C)^c \cap (B\cap C)^c$$

$$= (A^c\cap B^c)\cup C^c \qquad\qquad = (A^c\cup C^c)\cap(B^c\cup C^c)$$

$$= (A^c\cup C^c)\cap(B^c\cup C^c) \quad //$$





2) Reflexive: aRa, as $|a|=|a|$

Symmetric: Assume $aRb \Rightarrow |a|=|b|$

$bRa \Leftarrow |b|=|a|$

Transitive: Assume $aRb,\ bRc$

$\Rightarrow |a|=|b|,\ |b|=|c| \Rightarrow |a|=|c|$

$\Rightarrow aRc$