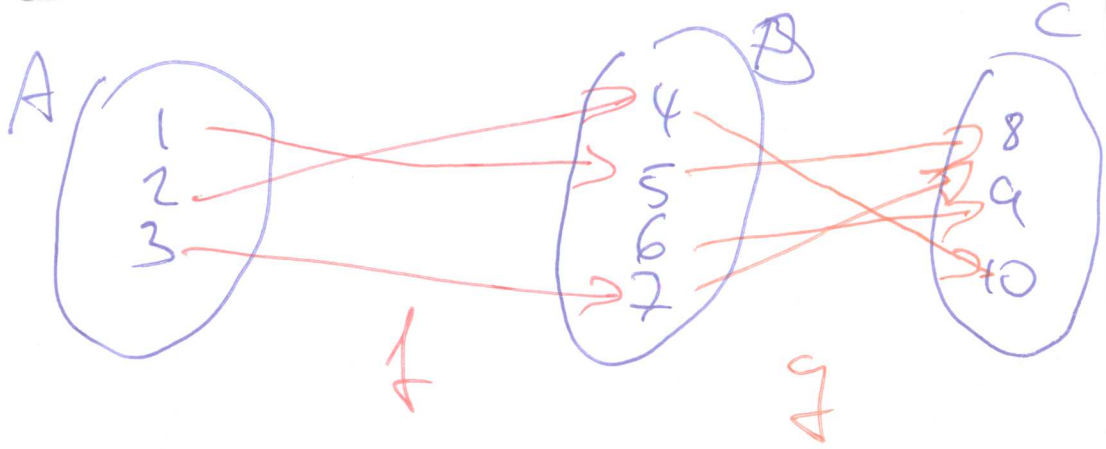
$f: A \to B$

$g: B \to C$

$g \circ f: x \mapsto g(f(x))$

$\uparrow$

A



$\exp(x+1) = (g \circ f)(x)$ for

$f(x) = x+1$, $g(x) = \exp(x)$

Obtaining: $(f \circ g): x \mapsto f(g(x))$

in this ex.: $\exp(x)+1$

Function $f: x \mapsto f(x)$

In Relation form:

$\{(x, f(x)) \mid x \in A\}$

Converse Relation:

$\{(f(x), x) \mid x \in A\}$

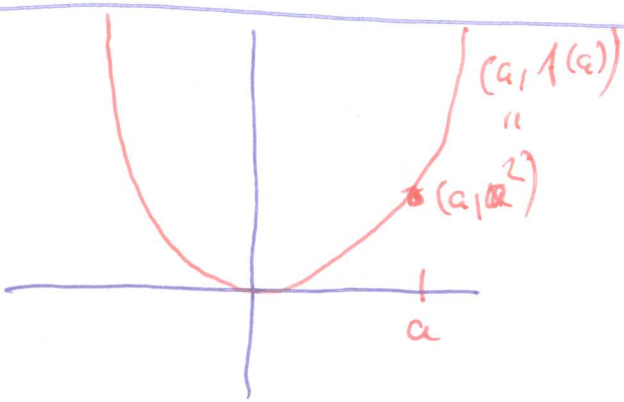
Converse is relation

$\{(x^2, x) \mid x \in \mathbb{R}\}$

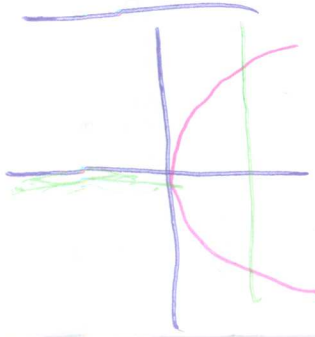
ex:

$f: \mathbb{R} \to \mathbb{R}$

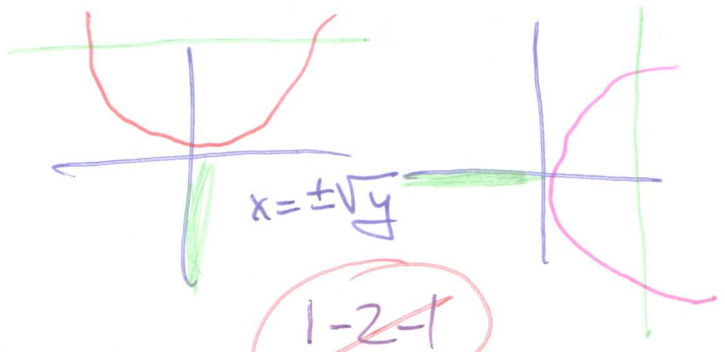
$x \mapsto x^2$



Converse:



When is the inverse of function f again a function.



vertical line test for inverse $\Leftrightarrow$ horizontal line test for f

1-2-1

Definition: Function f is **one-to-one** ("injective")
if $(f(a) = f(b) \Rightarrow a = b)$

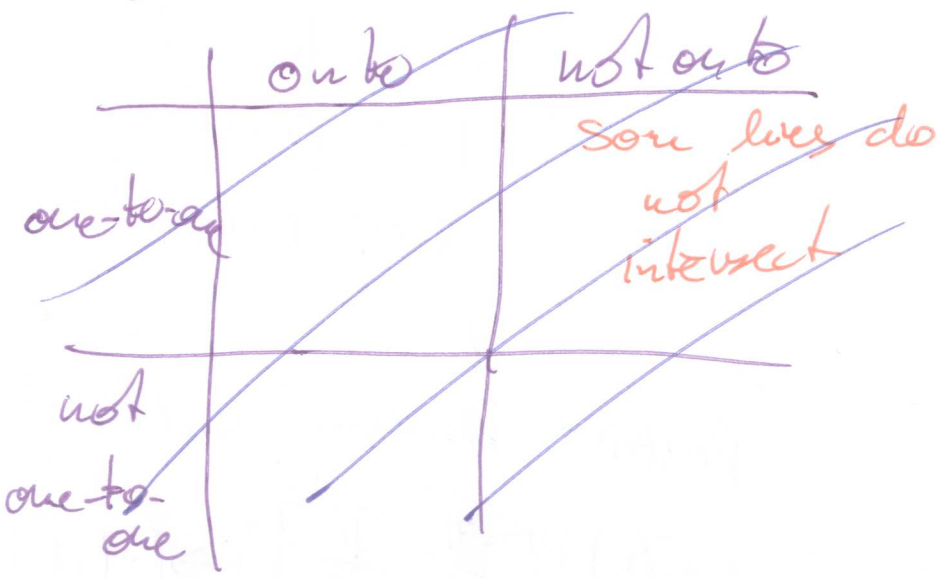
To test whether f given by formula is one-to-one
Solve $y = f(x)$ for x

for any x there is a value $\Rightarrow$ every possible value y is obtained as an image

Definition: Function f is **onto** ("surjective")
if $\forall b \in B$ there exists $a \in A$:
$f(a) = b$

Inverse of f is a function $\Leftrightarrow$ f is onto and one-to-one

Horizontal line intersects Negraph

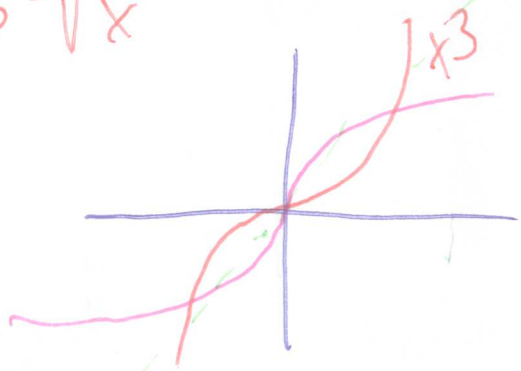


Eg $f: x \mapsto x^3 = y$.

Inverse fct: Solve for x.

$x = \sqrt[3]{y}$

$f^{-1}: x \mapsto \sqrt[3]{x}$ (Usually $x \Leftrightarrow y$)



onto: any line intersects (at least once)

not onto: (At least) one line does not intersect

one-to-one: any line intersects at most once

not-one-to-one: A line intersects twice (or more)

If f onto and one-to-one, then f has an inverse (=converse)

written f^{-1} ← Does not mean $\frac{1}{f}$

write $f^{-1}(x)$ inverse fct

$(f(x))^{-1} = \frac{1}{f(x)}$

What if f is not onto / not one-to-one

Eg: $\sin(x)$

$\sin(x): \mathbb{R} \to \mathbb{R}$

NOT ONTO

NOT ONE-TO-ONE

⇓

Restrict Domain and Range

$\sin(x): \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] \to [-1,1]$

⇐

Inverse: inverse sin or

$\arcsin: [-1,1] \to \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

IS ONTO and ONE-TO-ONE