11] $A = \{1, \dots, 20\}$, Relation P

$P = \{ (a,b) \in A \times A \mid$ a and b have same set of prime factors $\}$

This is equiv rel:

- reflexive : Show aPa, as a has same set of prime factors as a
- symmetric : Assume aPb, show bPa

  a has same prime factors as b

  $\Rightarrow$ b has — " ——— " — a
- transitive

Assume aPb and bPc. Show: aPc

a has same p.f. as b

b — " ——— c

$\Rightarrow$ a ——— " ——— c

Equivalence Classes:

$[1] = \{1\}$ | $[10] = \{10, 20\}$

$[2] = \{2, 4, 8, 16\}$ $[15] = \{15\}$

$[3] = \{3, 9\}$ | $[14] = \{14\}$

$[5] = \{5\}$, $[7] = \{7\}$, $[11] = \{11\}$

$[6] = \{6, 12, 18\}$ | $[13], [17], [19]$

13] Relation $R \subset \mathbb{Q} \times \mathbb{Q}$: $aRb \overset{\text{def}}{\Longleftrightarrow} \text{round}(a) = \text{round}(b)$

- reflexive : aRa, as $\text{round}(a) = \text{round}(a)$
- symmetric: $aRb \Rightarrow \text{round}(a) = \text{round}(b)$

  $\Rightarrow \text{round}(b) = \text{round}(a) \Rightarrow bRa$
- transitive aRb, bRc

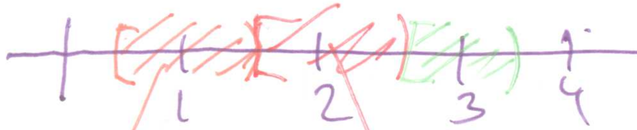
  $\hookrightarrow \text{round}(a) = \text{round}(b)$

  $\hookrightarrow \text{round}(b) = \text{round}(c)$

  $\Rightarrow \text{round}(a) = \text{round}(c) \Rightarrow aRc$

$[1] = \{a \in \mathbb{Q} \mid aR1\} = \{a \in \mathbb{Q} \mid \frac{1}{2} \leq a < \frac{3}{2}\}$

$\text{round}(a) = \text{round}(1) = 1$



$[1]$ $\quad$ $[2] = \{a \mid \frac{3}{2} \leq a < \frac{5}{2}\}$

The classes are $\{[a] \mid a \in \mathbb{Z}\}$

c) Not the same: $1.4 \not R\, 1.6$, as $\text{round}(1.4) = 1 \neq 2 = \text{round}(1.6)$

but $|1.4 - 1.6| < \frac{1}{2}$