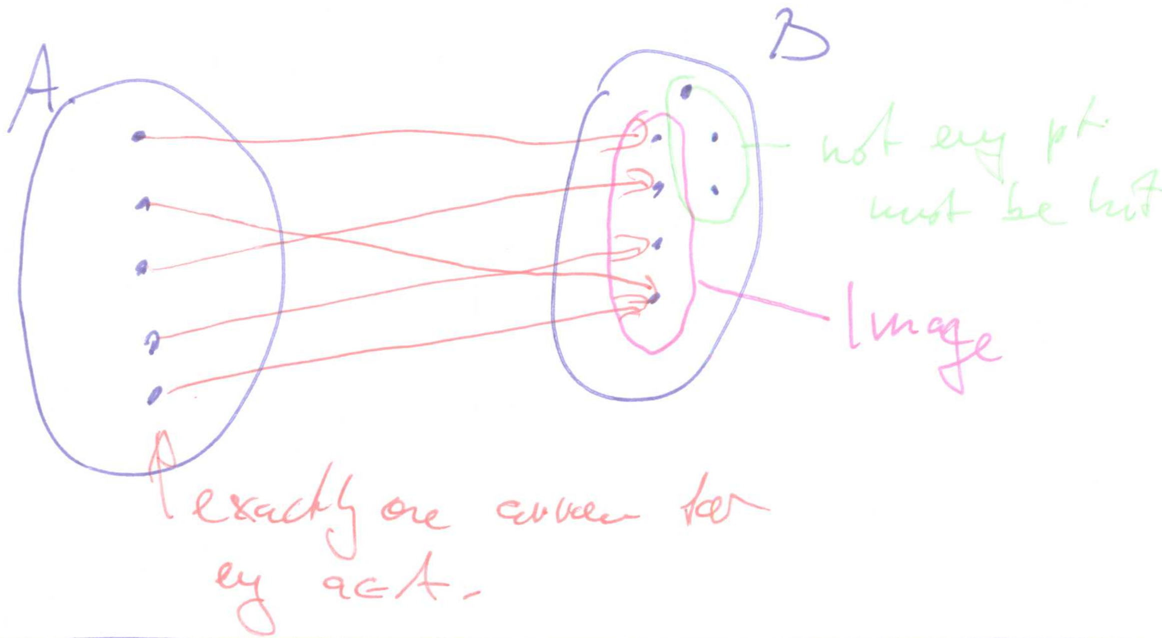
A function $f: A \to B$ is a relation $f \subseteq A \times B$ s.t.

a) $\forall a \in A: \exists\, b \in B: (a,b) \in f$ ← Source = Domain | A

b) $\forall\, (a,b), (a,c) \in f \Rightarrow b = c$

$\Rightarrow$ every $a \in A$ is related to exactly one $b \in B$:

$f(a) = b$, $f: a \mapsto f(a) = b$



Note: f function. $f(x)$ is the value of x under f

Ex: A = people
$B = \mathbb{Q}$
$f: A \to B$
$a \mapsto$ height of a in cm

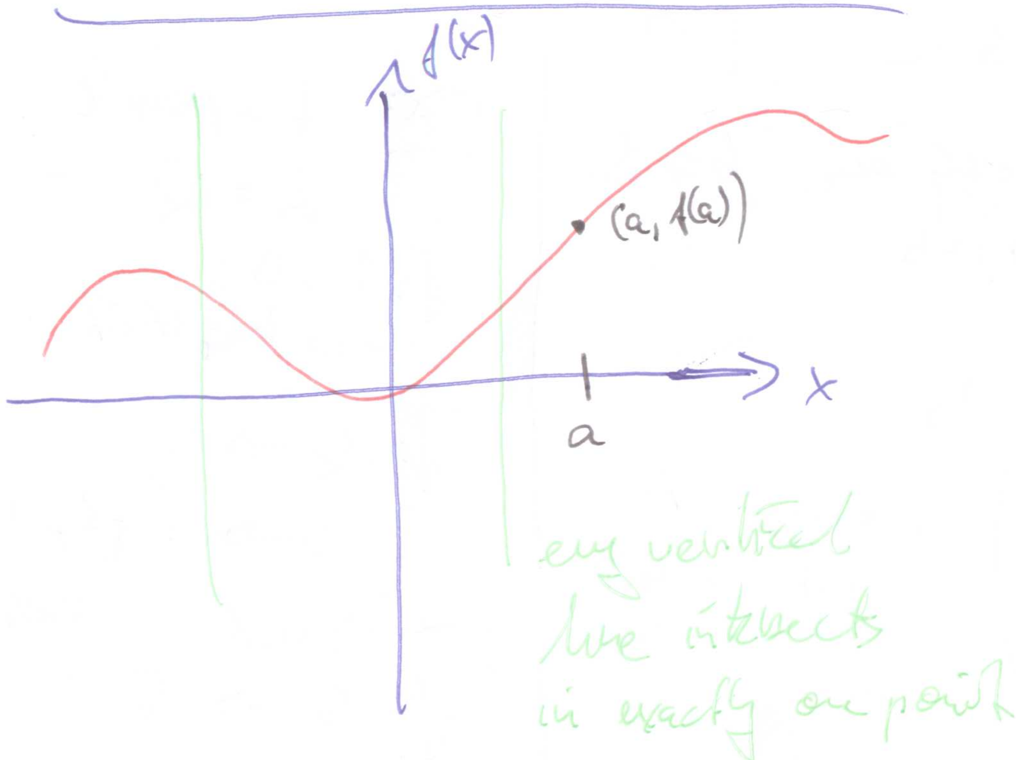
$f: \mathbb{Q} \to \mathbb{Q}$
$x \mapsto x^3 + 1$
argument value

$f: \mathbb{Q} \to \mathbb{Q}$

$$x \mapsto \begin{cases} x, & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$h: \mathbb{Q} \to \mathbb{R}$
$x \mapsto x^3 + 1$

Two functions are equal if
- same source
- same target
- $f(a) = g(a) \quad \forall a \in \text{Source}$



ex. $\mathrm{id} : A \to A$ "identity function"
$a \mapsto a$

constant function with value c: $A \to B \ni c$
$a \mapsto c$

Polynomial (Function):

$p : \mathbb{R} \to \mathbb{R}$

ex. $x \mapsto 5x^3 - 2x^2 + x + 1$

Generally:

$x \mapsto c_d x^d + c_{d-1} x^{d-1} + \cdots + c_1 x + c_0$ for $c_i \in \mathbb{R}$

$= \sum_{i=0}^{d} c_i \cdot x^i$

Rational Functions:

$f(x) = \dfrac{p(x)}{q(x)}$ for

polynomials p, q,

$q(x) \neq 0 \quad \forall x \in \text{Domain}$