$$((\underbrace{A \cap B^c}_{X}) \cup \underbrace{C^c}_{Y}) \cap \underbrace{(D \cup E)^c}_{Z}$$

$$(X \cup Y) \cap Z$$

$$= (X \cap Z) \cup (Y \cap Z)$$

Relations (Binary Relations)

Sets A, B

Relation

$R \subseteq A \times B$

$A = \{1,2,3\}$, $B = \{0,1,2\}$

Relation "is strictly smaller"

$\{(a,b) \in A \times B \mid a \lneq b\}$

$= \{(1,2)\}$

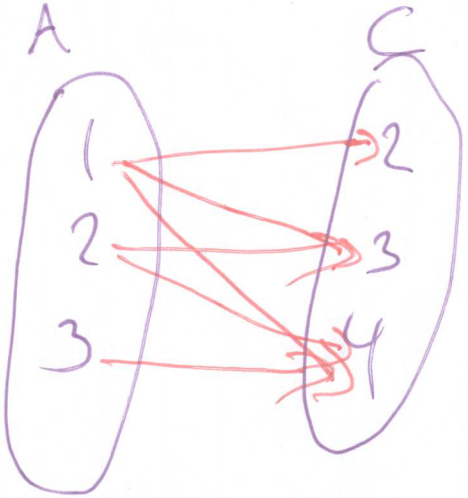
$C = \{2,3,4\}$

$\{(a,c) \in A \times C \mid a \lneq c\}$

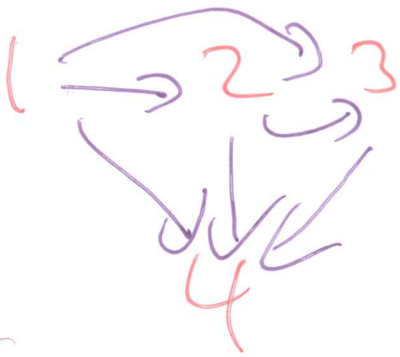
$= \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$

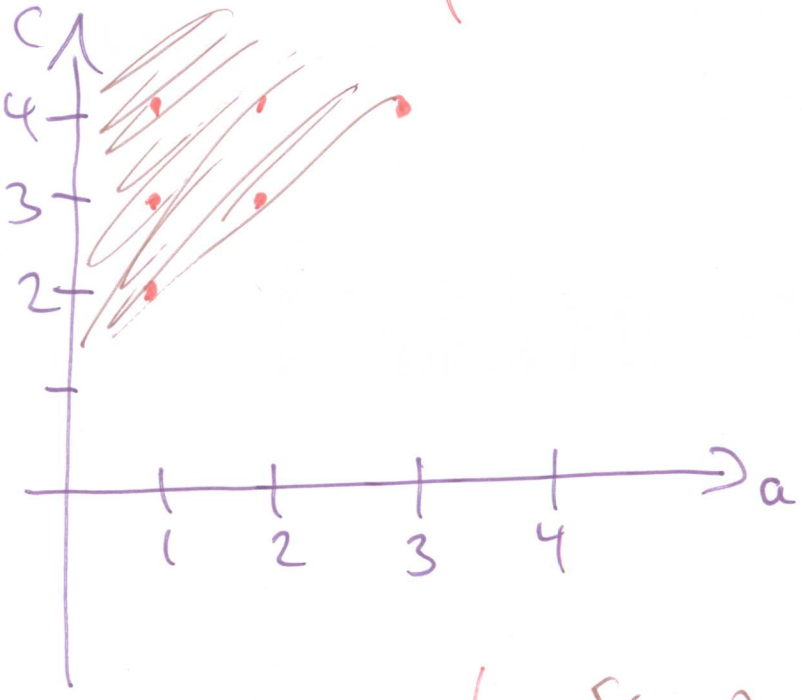
a	c
1	2
1	3
1	4
2	3
2	4
⋮	⋮

"Digraph"



Now $A = C = \{1,2,3,4\}$

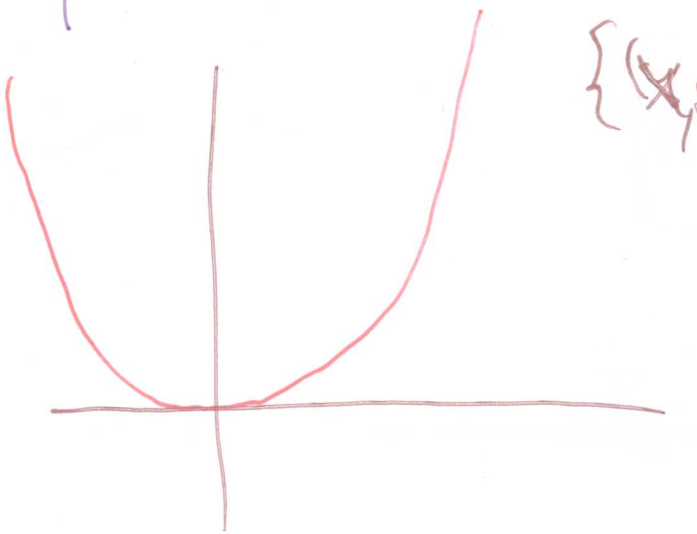




$(2,3) \in R$, $(3,2) \notin R$

$2 R 3$ $\quad$ $3 \not R 2$

$2 \leq 3$ $\quad$ $3 \neq 2$



$\{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = x^2\}$

If $R \subseteq A \times B$ (source $\in A$, target $\in B$)

Range $= \{b \in B \mid (a,b) \in R \text{ for some } a \in A\}$

Domain $= \{a \in A \mid (a,b) \in R \text{ for some } b \in B\}$

(Domain $\subseteq$ source)

$A = B = \{1, 2, 3, 4\}$

$R = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$

$R = \{(a,b) \mid a \leq b,\ a \in A, b \in B\} \subseteq A \times B$

Complement: $R^c = \{(a,b) \mid a \not R b\}$

$R^c = \{(2,1), (1,1), (2,2)\}$

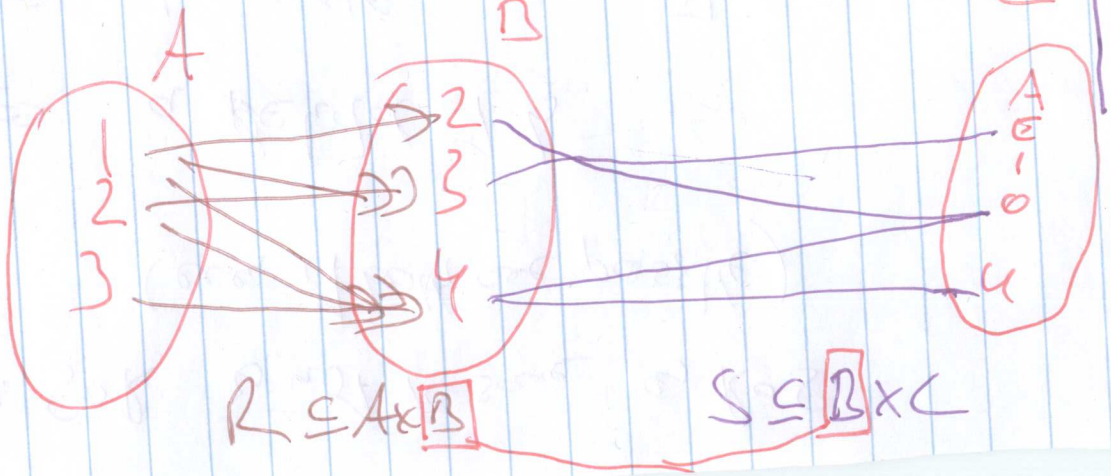
Converse: Converse of R (sometimes R^{-1})

$\subseteq B \times A$

$= \{(b,a) \in B \times A \mid a R b\}$

Converse of $R = \{(2,1), (3,1), (4,1), (3,2), (4,2), (4,3)\}$

Composition



$S \circ R = \{(1,0), (1,4), (1,4), (2,4), (2,0), (2,4), (3,0), (3,4)\}$

↑

Composition of R with S

Careful: $S \circ R$ is not the same as $R \circ S$
(even if both are possible)

$aRb \iff$ a child of b

$bSc \iff$ b is spouse of c

$S \circ R = \{(a,c) \mid$ a child of spouse of $c\}$

$R \circ S = \{$

(b,a) | b spouse of some c who is child of a