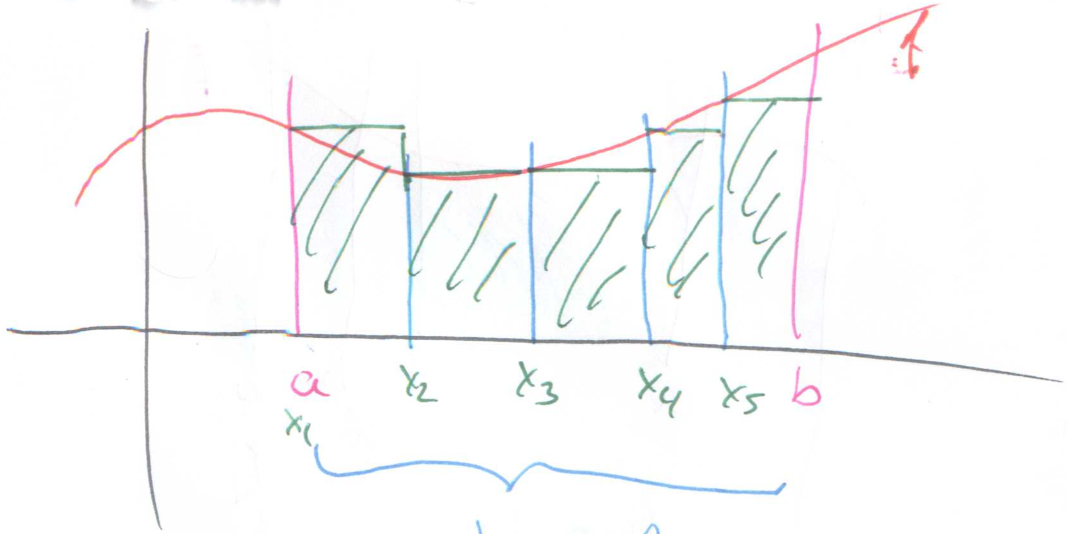


k parts

width $= \Delta x = \dfrac{(b-a)}{k}$

$$x_i = a + (i-1)\cdot\Delta x = a + (i-1)\frac{(b-a)}{k}$$

Area of rectangles:

$$A_k = \sum_{i=1}^{k} f(x_i)\cdot \Delta x$$

$$= \sum_{i=1}^{k} f\left(a + (i-1)\frac{(b-a)}{k}\right)\cdot \frac{(b-a)}{k}$$

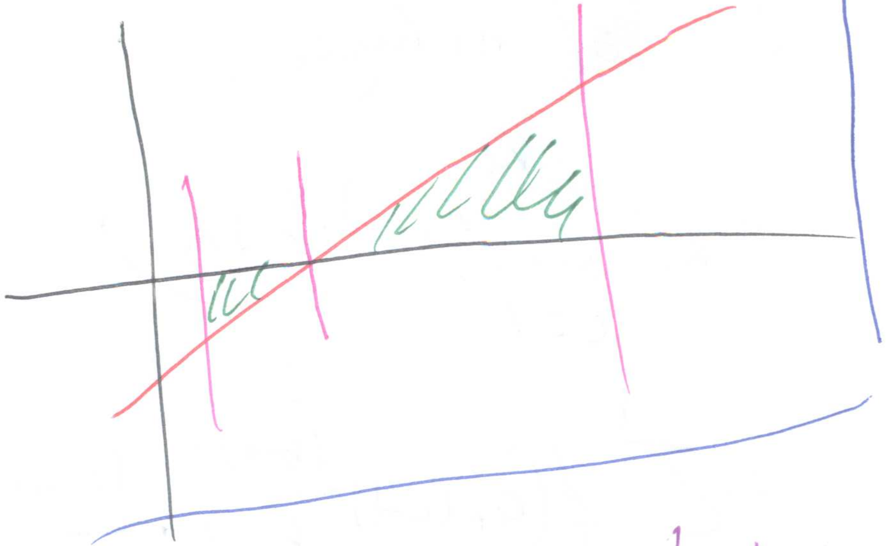
Riemann sums

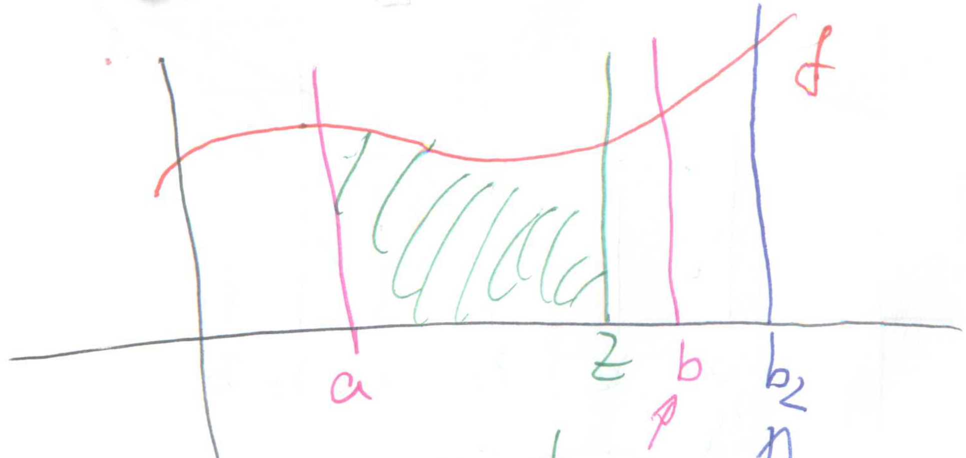
Def: The function f is integrable on the interval from a to b if $\lim\limits_{k\to\infty} A_k$ exists.

e.g. f is continuous

The value of the definite integral $\underbrace{\int_a^b f(x)\,dx}_{\text{area under graph}} = \lim\limits_{k\to\infty} A_k$

Riemann integral





Area $\int_a^b f(x)\,dx$

Area $\int_a^{b_2} f(x)\,dx$.

Define new function that gives the area underneath f from a=0 to $\cancel{x}$ z

$$F(\cancel{x}\,z) = \int_0^{\cancel{x}\,z} f(x)\,d\cancel{x}$$

$$\boxed{F(x)\Big|_a^b}$$

Any antiderivative G of f

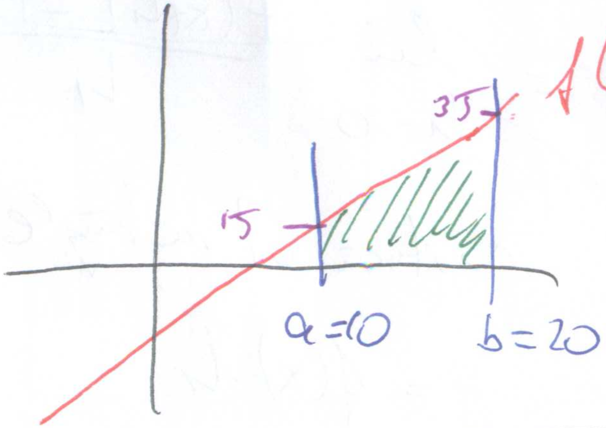
Theorem 11: (Fund Theorem of Calculus)

a) $\dfrac{d}{dz}F(z) = f(z)$. i.e. $F(x)$ is an antiderivative of f

b) For any $a,b \in \mathbb{R}$: $\int_a^b f(x)\,dx = \boxed{F(b) - F(a)} = G(b) - G(a)$

Ex:

$f(x) = 2x - 5$



$f(10) = 15$

$f(20) = 35$

$10 \cdot 15 = 150$

$+\frac{1}{2} \cdot 10 \cdot 20 = 100$

$= \boxed{250}$

$$\int_{10}^{20} 2x-5\,dx = x^2 - 5x\Big|_{10}^{20} = \underbrace{20^2 - 5\cdot 20}_{=300} - (10^2 - 5\cdot 10)$$

Fund Theorem

$= 300 - 50$

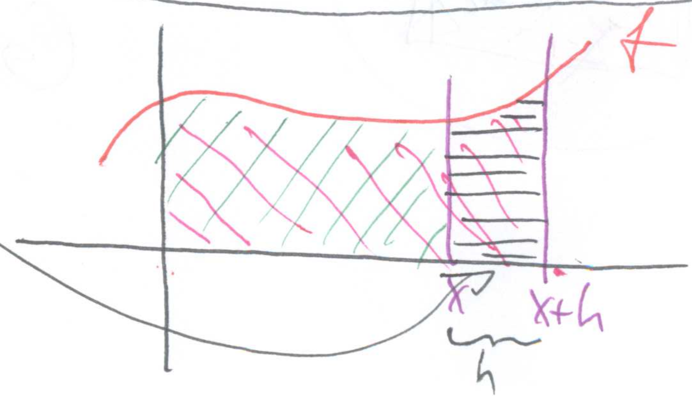
$= \boxed{250}$

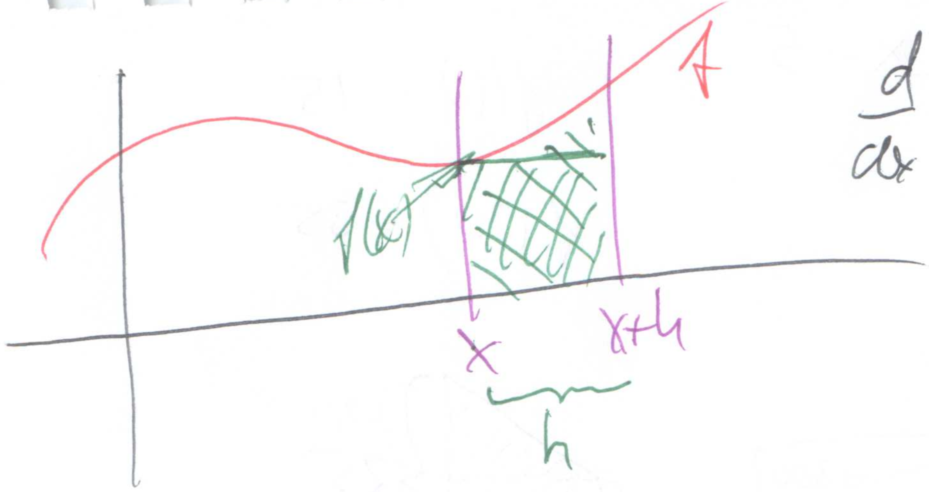
$$\int 2x-5\,dx = x^2 - 5x + C$$

Proof:

$$\frac{d}{dx}F(x) = \lim_{h\to 0}\frac{F(x+h) - F(x)}{h}$$

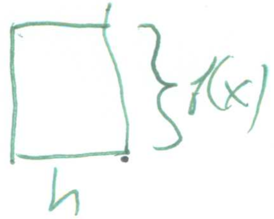
def. derivative





$$\frac{d}{dx}F(x) = \lim_{h\to 0}\left(\frac{F(x+h)-F(x)}{h}\right)$$

$\approx$ Area of rectangle



$$= f(x)\cdot h$$

$$= f(x) \Rightarrow \text{Area fct. is antider.}$$

b) $\int_a^b f(x)\,dx = \int_0^b f(x)\,dx - \int_0^a f(x)\,dx = F(b) - F(a)$

c) Antiderivative : $G(x) = F(x) + c$ ← constant;

$$G(b) - G(a) = F(b) + \cancel{c} - \left(F(a) + \cancel{c}\right)$$

$$= F(b) - F(a)$$