Substitution:

$$\int \cos(x^2)\,\frac{2}{2}\,\underset{x^2\cdot x}{x^3}\,dx = \int \frac{1}{2}\cos(u)\,u\cdot du = \frac{1}{2}\left(u\cdot\sin(u) - \int \sin(u)\,du\right)$$

$u = x^2$

$\frac{du}{dx} = 2x, \quad du = 2x\,dx$

$f = u, \quad g' = \cos(u)$

$f' = 1, \quad g = \sin(u)$

$$= \frac{1}{2}\left(u\sin(u) + \cos(u) + C\right)$$

$$= \frac{1}{2}\left(x^2\sin(x^2) + \cos(x^2)\right) + C$$

$$\int \sin(x)^5 \cdot \boxed{\cos(x)\,dx} = \int u^5\,du = \frac{1}{6}u^6 + C = \frac{1}{6}\sin(x)^6 + C$$

$\boxed{u = \sin(x)}$ My Choice

$du = \boxed{\cos(x)\,dx}$

$\frac{du}{dx} = \cos(x)$ (Derv. calc.) $\quad dx = \frac{du}{\cos(x)}$

$$\int \cos(x^2)\, x^2\, dx = \int \cos(u)\, \frac{1}{2}\, \cancel{x}\sqrt{u}\, du$$

$u = x^2 \Rightarrow x = \sqrt{u}$

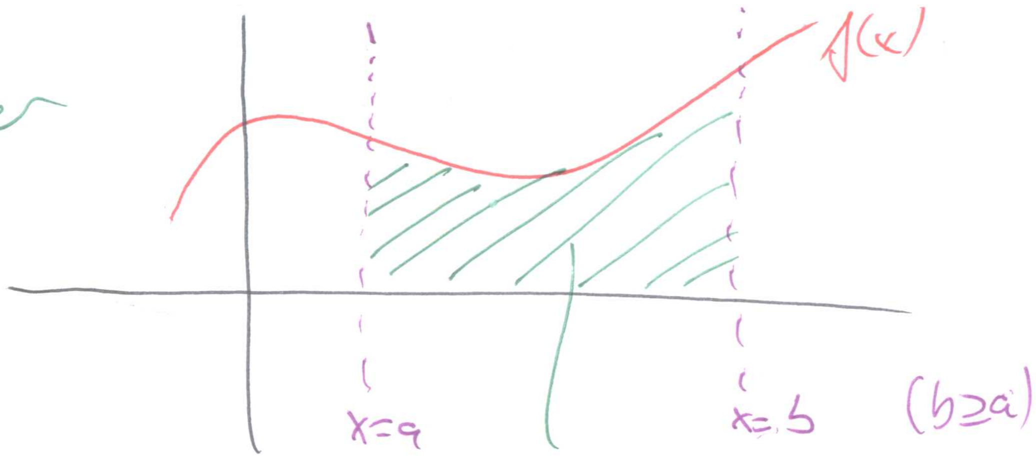
$du = 2x\, dx$

Definite Integrals

The definite integral ~~of~~ over $f(x)$ from a to b is

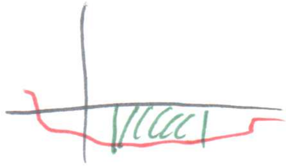
$$\int_a^b f(x)\, dx = \text{Area}$$

bounded by:
- x-axis
- graph of f
- lines $x=a$, $x=b$

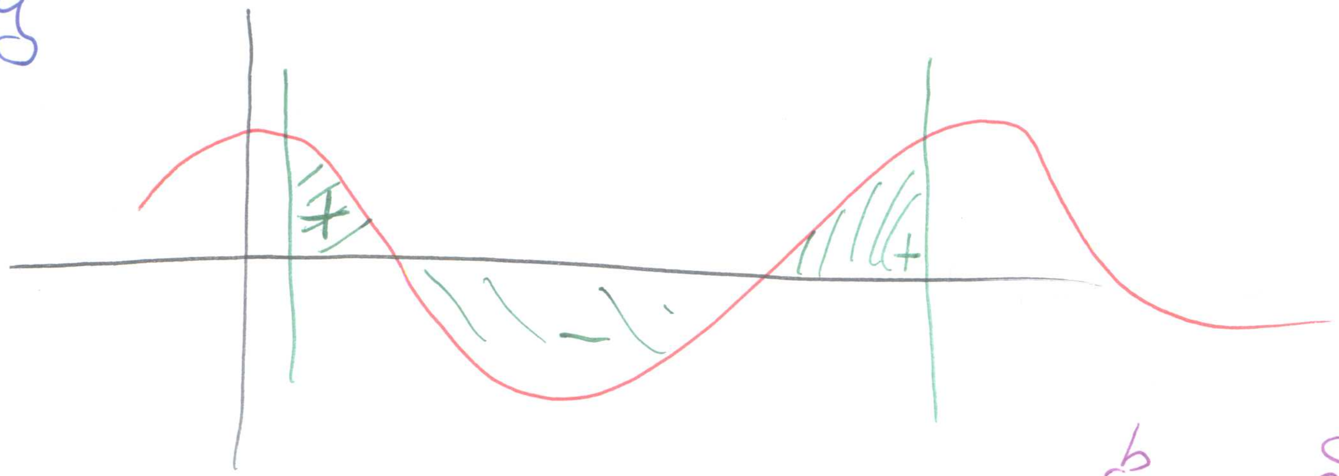


Area

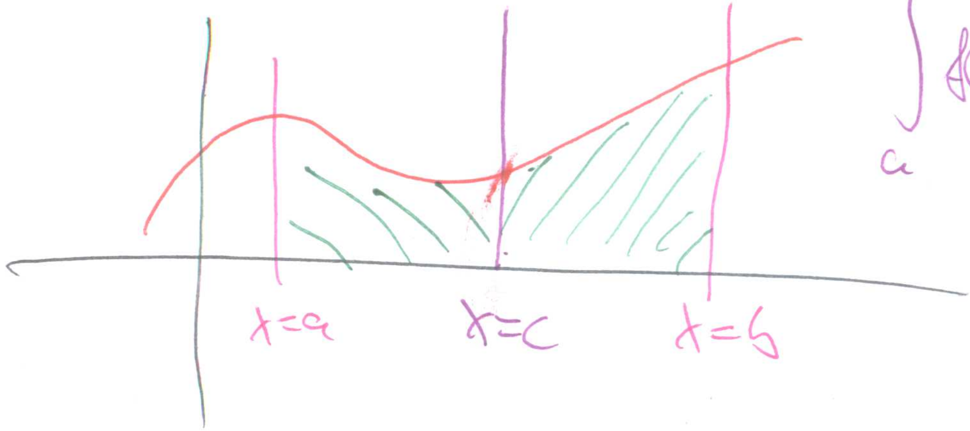
- $a=b$: $\int_a^a f(x)\, dx = 0$
- $b<a$: count negatively
- If $f(x) \le 0$: negative area



More generally



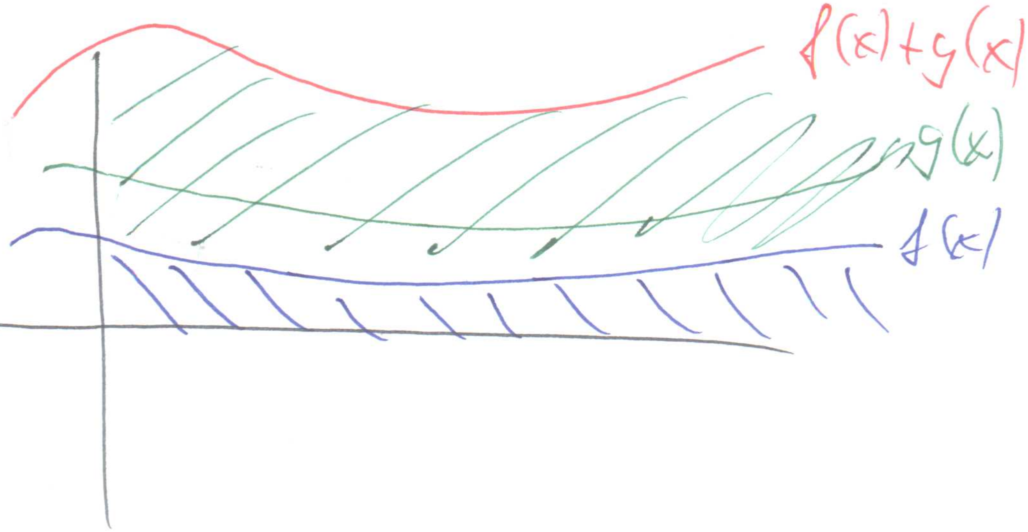
Split into two



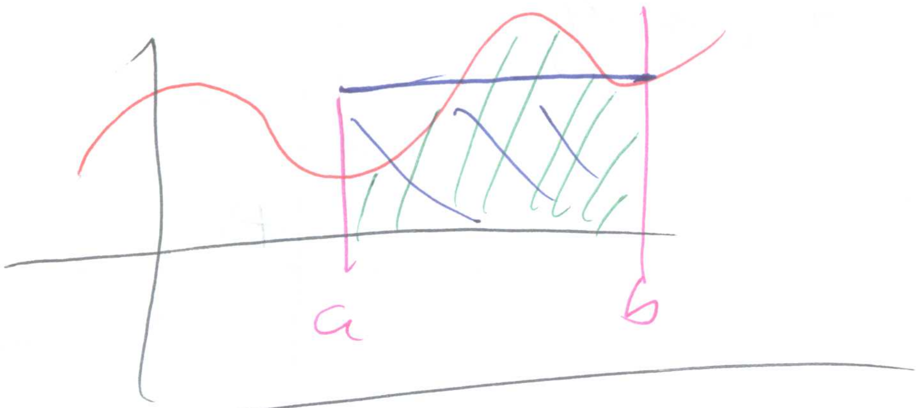
$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

Sum of fcts

$$\int_a^b f(x)+g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$



Example use: Averaging



Average value of $f(x)$

from a to b is

$$\left(\frac{\int_a^b f(x)\,dx}{b-a} \right)$$