Show: $x^2 \cdot \lg(x)^{-15x} = \Theta(x^3)$

Means: ex constants c, N: $x^2 \lg(x) \leq c \cdot x^3$ as long as $x \geq N$

Method 1: directly: As $\lg(x) \leq x$ $\forall x \geq 1$

N

we have: $x^2 \lg(x) \leq x^2 \cdot x = x^3$

$c = 1$

Method 2: Using L'Hospital:

$$\lim_{x \to \infty} \frac{\cancel{x^2} \lg(x)}{x^{\cancel{3}}} = \lim_{x \to \infty} \frac{\lg(x)}{x} \overset{\text{L'Hospital}}{=} \lim_{x \to \infty} \frac{1/x}{1} = \lim_{x \to \infty} \frac{1}{x} = 0$$

Numerator $= \Theta$(denom)

$F(x)$

$\frac{d}{dx}$

$f(x) = F'(x)$

$\int f(x)\,dx = F(x) + C$

$\int f(x)\,dx$

$f(x)$

Ex: $\int x^5 - 7x + 1 \, dx = \frac{1}{6}x^6 - \frac{7}{2}x^2 + \frac{1 \cdot x}{1} + C$

Ex: $\int x^2 \cdot \sin(x) \, dx = -\cos(x) \cdot x^2 + \int 2x\cos(x) \, dx = -\cos(x) \cdot x^2 + 2x \cdot \sin(x)$

$f(x) = x^2, \quad g'(x) = \sin(x)$

$f'(x) = 2x, \quad g(x) = -\cos(x)$

$f(x) = x, \quad g'(x) = \cos(x)$

$f'(x) = 1 \quad g(x) = \sin(x)$

$-2\int \cos(x) \, dx$

$= -2\sin(x) + C$

$= -\cos(x) \cdot x^2 + 2x \cdot \sin(x) - 2\sin(x) + C$

Ex: $\int \log(x) \cdot dx = x \cdot \log(x) - \int \cancel{\frac{x}{x}}^{\,1} dx = x \cdot \log(x) - x + C$

$f(x) = \log(x), \quad g'(x) = 1$

$f'(x) = \frac{1}{x}, \quad g(x) = x$

Verify: $\frac{d}{dx}\left(x\log(x) - x\right)$

$= 1 \cdot \log(x) + \cancel{x \frac{1}{x}} - \cancel{1} = \log(x)$

Chain Rule: $\frac{d}{dx} f(g(x)) = \cancel{f'(x)}$
$f'(g(x)) \cdot g'(x)$

Antiderivatives: $\int f'(\underbrace{g(x)}_{u=g(x)}) \cdot \underbrace{\boxed{g'(x)\,dx}}_{du} = f(g(x)) + C$

Substitution

Then $\frac{du}{dx} = \frac{d}{dx} g(x) = g'(x) \Rightarrow du = \boxed{g'(x)\,dx}$

or $dx = \frac{du}{g'(x)}$

$= \int f'(u)\,du = f(u) + C = f(g(x)) + C$

Ex: $\int (\sin(x))^2 \boxed{\cos(x)\,dx} = \int u^2\,du = \frac{1}{3}u^3 + C = \frac{1}{3}(\sin(x))^3 + C$

$u = \sin(x)$.
$du = \boxed{\cos(x)\,dx}$.

Ex: $\int (\sin(x))^2 dx = \int u^2 \cdot \frac{1}{\cos(\arcsin(u))} \cdot du$?

$u = \sin(x)$

$du = \cos(x) dx$

$x = \arcsin(u)$ $dx = \frac{du}{\cos(x)}$

$= \frac{du}{\cos(\arcsin(u))}$

no occurrence of x left.

$\int \frac{2}{2} \cancel{\cos(x^2)\, x}\, \cos(x^2) \cdot x \, dx = \int \frac{1}{2} \cos(u) du = \frac{1}{2} \sin(u) + C = \frac{1}{2} \sin(x^2) + C$

$u = x^2$

$du = 2x dx$

a) Taylor Pol for sin(x), deg 5, around $a=0$

$-5 \quad 7$

$\boxed{-1 \le x \le 1}$

$-1 \quad a=0 \quad 1$

$\ell = |x-a| \le 1$

$$\text{error} \le \frac{M}{6!}\,\ell^6$$

Max. value of 6-th derivative on interval

$$\le \frac{1}{6!}\cdot 1^6 = \frac{1}{720} \qquad M=1$$

b) log(x) around $a=2$, $1 \le x \le 5$ deg 4

$\ell = |x-a| \le 3$

$$\text{Error} \le \frac{M}{5!}\cdot 3^5 \qquad M=2$$

$$= \frac{2}{120}\cdot 243 = \frac{81}{20} \sim 0.26\ldots$$