

Show: $x^2 \cdot \log(x) = O(x^3)$

Means: ex constant: $x^2 \log(x) \leq c \cdot x^3$ as $\log$ as $x \geq N$
 c, N

Method 1: directly: As $\log(x) \leq x$ $\forall x \geq 1$
 we have: $x^2 \log(x) \leq x^2 \cdot x = x^3$
 $c=1$

Method 2: Using L'Hospital:

$$\lim_{x \rightarrow \infty} \frac{x \log(x)}{x^3} = \lim_{x \rightarrow \infty} \frac{\log(x)}{x} \stackrel{\text{L'Hospital}}{=} \lim_{x \rightarrow \infty} \frac{1/x}{1} = \lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

$$\begin{array}{c} F(x) \\ \downarrow \frac{d}{dx} \\ A(x) = F'(x) \end{array}$$

$$\int f(x) dx = F(x) + C \quad \text{Inverse} = O(\text{derivative})$$

$$\uparrow \int f(x) dx$$

$$f(x)$$

Ex: $\int x^5 - 7x + 1 \, dx = \frac{1}{6} x^6 - \frac{7}{2} x^2 + \frac{1}{1} x + C$

Ex: $\int x^2 \cdot \sin(x) \, dx = -\cos(x) \cdot x^2 + \int 2x \cos(x) \, dx = -\cos(x) \cdot x^2 + 2x \sin(x)$

$f(x) = x^2, g'(x) = \sin(x)$

$f'(x) = 2x, g(x) = -\cos(x)$

$f(x) = x, g'(x) = \cos(x)$

$f'(x) = 1, g(x) = \sin(x)$

$-2 \int \cos(x) \, dx = -2 \sin(x) + C$

$= -\cos(x) \cdot x^2 + 2x \cdot \sin(x) - 2 \sin(x) + C$

Ex: $\int \log(x) \cdot dx$

$= x \cdot \log(x) - \int \frac{x}{x} \, dx = x \cdot \log(x) - x + C$

$f(x) = \log(x), g'(x) = 1$

$f'(x) = \frac{1}{x}, g(x) = x$

Verif: $\frac{d}{dx} (x \log(x) - x)$

$= 1 \cdot \log(x) + x \cdot \frac{1}{x} - 1 = \log(x)$

Chain Rule: $\frac{d}{dx} f(g(x)) = \cancel{f'(x)} f'(g(x)) \cdot g'(x)$

Antiderivatives: $\int \underbrace{f'(g(x))}_{u=g(x)} \cdot \boxed{g'(x) dx} = \underline{\underline{f(g(x)) + C}}$

Substitution

Per $\frac{du}{dx} = \frac{d}{dx} g(x) = g'(x) \Rightarrow du = \boxed{g'(x) dx}$
 or $dx = \frac{du}{g'(x)}$

$= \int f'(u) du = f(u) + C = \underline{\underline{f(g(x)) + C}}$

Ex: $\int (\sin(x))^2 \cdot \boxed{\cos(x) dx} = \int u^2 du = \frac{1}{3} u^3 + C = \frac{1}{3} (\sin(x))^3 + C$

$u = \sin(x)$
 $du = \boxed{\cos(x) dx}$

Ex: $\int (\sin(x))^2 dx = \int u^2 \cdot \frac{1}{\cos(\arcsin(u))} du$ ✗ ?

$u = \sin(x)$

$du = \cos(x) dx$

$x = \arcsin(u) \quad dx = \frac{du}{\cos(x)}$
 $= \frac{du}{\cos(\arcsin(u))}$

no occurrence
of x left.

$\int \frac{2 \cos(x^2) \cdot x}{2} dx = \int \frac{1}{2} \cos(u) du = \frac{1}{2} \sin(u) + C = \frac{1}{2} \sin(x^2) + C$

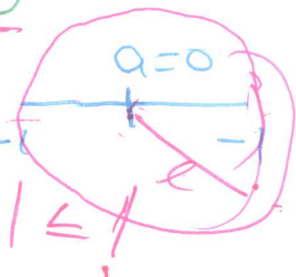
$u = x^2$

$du = 2x dx$

a) Taylor pol for $\sin(x)$, deg 5,

around $a=0$

$$-5 \leq x \leq 5$$



$$\text{Error} \leq \frac{M}{6!} l^6$$

Max. value of 6-th derivative on interval

$$\leq \frac{1}{6!} \cdot 1^6 = \frac{1}{720}$$

$$M=1$$

b) $\lg(x)$ around $a=2$, $1 \leq x \leq 5$ deg 4

$$\text{Error} \leq \frac{M}{5!} \cdot 3^5$$

$$l = |x-a| \leq 3$$

$$M=2$$

$$= \frac{2}{120} \cdot 243 = \frac{81}{20} \sim 0.26 \dots$$