Differentiation: $f(x)$ $\xrightarrow{\frac{d}{dx}}$ $f'(x) = \frac{d}{dx} f(x)$

Antidifferentiation / Integration: $h(x) + C$ $\xleftarrow{\ \ }$ $g(x) = \frac{d}{dx}$ $\int \underbrace{g(x)}_{\text{Integrand}} dx$

Foundations

Ex: $\int \underline{\cos(x)}\, dx = \sin(x) + C$ (as $\frac{d}{dx} \sin(x) = \cos(x)$)

$\int \underline{\exp(x) \cdot (x+1)}\, dx = x \cdot \exp(x) + C$ (as $\frac{d}{dx}\, x \cdot \exp(x) = \exp(x) \cdot (x+1)$)

$\frac{d}{dx} \left(f(x) + g(x) \right) = f'(x) + g'(x)$

For c constant:

$\frac{d}{dx} \left(c \cdot f(x) \right) = c \cdot f'(x)$

a constant

$\frac{d}{dx} x^a = a \cdot x^{a-1}$

$\int f(x) + g(x)\, dx = \int f(x)\, dx + \int g(x)\, dx$

$\int c \cdot f(x)\, dx = c \cdot \int f(x)\, dx$

$\int x^a\, dx = \frac{1}{a+1} x^{a+1} + C$

$a \neq -1$

$\frac{d}{dx} \log(x) = \frac{1}{x}$

$\int x^{-1} dx = \log(x) + C$

Ex: $\int \boxed{x^3} + \boxed{3x} + \boxed{7} dx = \underbrace{\frac{1}{4}x^4}_{} + \frac{3}{2}x^2 + 7x + C$

$\frac{d}{dx} \frac{1}{4}x^4 = \frac{4}{4}x^3 = x^3$

Product Rule

$\frac{d}{dx}\left(f(x) \cdot g(x)\right) = \underline{f'(x) \cdot g(x)} + f(x) \cdot g'(x)$

$\int f'(x) \cdot g(x) dx = \int \frac{d}{dx} \left(f(x) \cdot g(x)\right) dx - \int f(x) \cdot g'(x) dx$

Verify: $\frac{d}{dx}\left(\sin(x) \cdot x + \cos(x)\right) = \cos(x) \cdot x + \cancel{\sin(x)} \cancel{- \sin(x)}$

Integration by parts

$$\boxed{\int f'(x) \cdot g(x) dx = f(x) \cdot g(x) - \int f(x) \cdot g'(x) dx}$$

Ex: $\int \cos(x) \cdot x \, dx = \sin(x) \cdot x - \int \sin(x) dx = \sin(x) \cdot x + \cos(x) + C$

$f' = \cos(x)$, $g(x) = x$

$f = \sin(x)$, $g'(x) = 1$

$$\int \exp(x)\cdot x\,dx = \exp(x)\cdot x - \underbrace{\int \exp(x)\cdot 1\,dx} = \exp(x)\cdot x - \exp(x) + C$$

$$= \exp(x)(x-1) + C$$

$f'(x) = \exp(x)$, $g(x) = x$

$f(x) = \exp(x)$, $g'(x) = 1$

Other choice:

$\cancel{f(x) = x,\ g'(x) = \exp(x)}$

$f'(x) = x$, $g(x) = \exp(x)$

$f(x) = \frac{1}{2}x^2$, $g'(x) = \exp(x)$

$$= \frac{1}{2}x^2\cdot\exp(x) - \int \frac{1}{2}x^2\exp(x)\,dx$$

more complicated