Homotopy theory for graphs
Laura Scull
Fort Lewis College, Durango, CO

Homotopy theory studies deformations of space, an inherently continuous concept. In this talk, we will explore how to translate this concept to the discrete category of graphs. We will see how to define homotopy for graphs by using categorical concepts, and then show how to analyze what these translate to concretely, and how we can break down a deformation of graph maps into moves of one vertex at a time. We also discuss how to recognize when two graphs are the same up to deformation (“homotopy equivalent”), and look at developing homotopy invariants for graphs. No prior knowledge of either homotopy theory or graph theory will be needed, and many examples will be given. Parts of this work are joint with Dr. Tien Chih at MSU Billings, and parts were developed in collaboration with Fort Lewis College undergraduate students Coleman Kane, Diego Novoa and Jonathon Thompson.

Combinatorics and Group Symmetry in [Hilbert Space] Frame
Emily King
Colorado State University

Frames are collections of vectors in Hilbert spaces which have reconstruction properties akin to orthonormal bases. In order for such a representation system to be robust in applications, one often asks that the vectors be geometrically spread apart; that is, the pairwise angles between the lines they span should be as large as possible. It ends up that structures in algebraic combinatorics, like difference sets and balanced incomplete block designs (BIBDs), can be used in different ways to construct optimal configurations. Furthermore, the linear dependencies of the vectors are often encoded as BIBDs. The orbit of a vector under a group actions sometimes also yields optimal configurations. There is at least one infinite class of frames, the so-called Gabor-Steiner ETFs which have both group symmetry and a combinatorial construction. In this talk, these and other connections between frames and algebraic combinatorics, combinatorial design theory, algebraic graph theory, and more will be presented. A couple of open conjectures in frame theory and quantum information theory will also be discussed, with one in particular possibly being amenable to combinatorial methods. Frames, difference sets, BIBDs, and affine geometry will be defined and explained, making at least part of the talk accessible to a more general audience.