Mathematics Seminar

Rocky Mountain Algebraic Combinatorics Seminar

Structure in sumsets I

John Griesmer
Colorado School of Mines

Given two subsets A, B of an abelian group, their *sumset* is $A + B := \{a + b :\in A, b \in B\}$. Their study is divided roughly into *direct theorems*, which deduce properties of $A+B$ from hypotheses on A and B, and *inverse theorems*, which deduce properties of A and B from hypotheses on A and B.

A prototypical direct theorem is the Cauchy-Davenport theorem, which states that if p is prime and A, B are nonempty subsets of $\mathbb{Z}/p\mathbb{Z}$, then $|A+B| \geq \min\{|A|+|B|-1, p\}$. The corresponding inverse theorem, due to A. G. Vosper, provides a simple classification of the pairs A, B where equality occurs.

Another classical result is the Steinhaus lemma, which says that if $A, B \subset \mathbb{R}$ both have positive Lebesgue measure, then $A + B$ contains an interval.

We will survey these and other classical results, together with their proof techniques, including the basic theory of Fourier analysis as it applies to sumsets. We will also discuss connections to other results in additive combinatorics, such as the recent Kelley-Meka breakthrough on counting three-term arithmetic progressions in sets of integers. No background in measure theory is required.

Structure in sumsets II

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Given two subsets A, B of an abelian group, their *sumset* is $A + B := \{a + b :\in A, b \in B\}$. An *inverse theorem* on sumsets derives structural results on A and B from hypotheses on $A + B$. For example, if A and B are nonempty sets of integers with $|A+B| < |A|+|B|$, then A and B are arithmetic progressions with the same common difference, or $|A| = 1$ or $|B| = 1$. We will survey some of the many recent results determining the structure of pairs A and B where $|A + B|$ is "slightly larger" than $|A| + |B|$ (in various senses).

This will motivate the following general problem: how can one construct sets A and B where $A + B$ lacks some prescribed structure? For example, given $K, N \in \mathbb{N}$, how can one construct a set $A \subset (\mathbb{Z}/2\mathbb{Z})^N$ with $|A| \approx \frac{1}{2}2^N$, where $A+A$ contains no subgroup of index at most K? Igor Kriz and Imre Ruzsa independently developed the same construction of such an example, variations of which are the only known way large subsets of finite abelian groups whose sumset lacks some prescribed structure. One aim of this talk is to advertise the problem of either finding a fundamentally different example, or proving that all such examples are essentially the same as Kriz and Ruzsa's.

Weber 223
4–6 pm, Friday, March 3, 2023
(Refreshments 3:30–4 pm)
Colorado State University
4 pm, Friday, March 3, 2023

This is a joint Denver U / UC Boulder / U of Wyoming / CSU seminar that meets biweekly.
Anyone interested is welcome to join us at a local restaurant for dinner after the talks.

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