

Sample Exam 1 for Math130

Use the following preference schedule to answer questions #1 - #10:

# of Voters	9	2	10	5	7
1 st	A	D	A	B	D
2 nd	B	B	C	C	B
3 rd	C	A	D	D	C
4 th	D	C	B	A	A

1) How many first-place votes does candidate A receive?

A receives 19 first place votes. ← $19 = 9 + 10$
 B receives 5 first place votes. ←
 C receives 0 first place votes. ←
 D receives 9 first place votes. ← $9 = 2 + 7$

2) What is the minimum number of first-place votes needed to have a majority?

Find (total number of first place votes)/2. If you get a whole number, the majority is the next whole number. If you get a decimal, round up (no such thing as 1/2 a vote).

$$\frac{33}{2} = 16.5 \quad \boxed{17}$$

another example
 $\frac{34}{2} = 17 \quad \boxed{18}$

3) Which candidate has a majority of the first-place votes?

Ask: Do any of the candidates have 17 or more first-place votes? **A**

4) Rank the candidates using the plurality method.

All that matters is how many first-place votes each candidate gets. In a ranked election, the candidate with the most first-place votes is first, the candidate with the second most first-place votes is second, and so on.

A, D, B, C

Use the following preference schedule to answer questions #1 - #10:

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3 rd	C	A	D	D	C
4 th	D	C	B	A	A

5) How many points does candidate B earn when using the method of pairwise comparisons?

B vs. A:

B is preferred over A $\frac{2+5+7=14}{14}$ times.

A is preferred over B $\frac{9+10=19}{19}$ times.

} 1pt. for A

B vs. C:

B is preferred over C $\frac{23}{23}$ times.

C is preferred over B $\frac{10}{10}$ times.

} 1pt for B

B vs. D:

B is preferred over D $\frac{14}{14}$ times.

D is preferred over B $\frac{19}{19}$ times.

} 1pt for D

1

6) Which candidate wins using the method of pairwise comparisons?

Building on 5), include the following:

A vs. C

A is preferred over C $\frac{21}{21}$ times.

C is preferred over A $\frac{12}{12}$ times.

} 1pt for A

A vs. D

A is preferred over D $\frac{19}{19}$ times.

D is preferred over A $\frac{14}{14}$ times.

} 1pt. for A

C vs D

C is preferred over D $\frac{24}{24}$ times.

D is preferred over C $\frac{9}{9}$ times.

} 1pt. for C

Conclusion:

A: 3 point(s).

B: 1 point(s).

C: 1 point(s).

D: 1 point(s).

7) Which candidate is a Condorcet candidate?

A

The Condorcet candidate beats ALL of the other candidates in pairwise comparisons. The method of pairwise comparisons always chooses the Condorcet candidate (if there is one) as the winner of the election.

Candidate A is a Condorcet candidate.

Rewriting the preference schedule helps.

Number of Voters	9	2	10	5	7
1 st	4 pts A (36)	D (8)	A (40)	B (20)	D (28)
2 nd	3 pts B (27)	B (6)	C (30)	C (15)	B (21)
3 rd	2 pts C (18)	A (4)	D (20)	D (10)	C (14)
4 th	1 pt D (9)	C (2)	B (10)	A (5)	A (7)

8) How many points does candidate B earn using the Borda count method?

$$27 + 6 + 10 + 20 + 21 = \boxed{84}$$

9) Using the Borda count method, which candidate comes in 2nd place?

A: $36 + 4 + 4 + 5 + 7$ points = 92

B: 84 points

C: 79 points

D: 75 points

B

10) Is the Majority Criterion violated with this preference schedule? **NO**

The Majority Criterion: A candidate with a majority of the first-place votes should be the winner.

Do any of the candidates have a majority of the first-place votes? **A**

If so, does this candidate win under each method? yes

Use the following preference schedule to answer #11:

Number of Voters	5	26	18	6	11	3
1 st	B	C	B	A	C	A
2 nd	C	B	A	C	B	B
3 rd	A	D	D	D	A	C
4 th	D	A	C	B	D	D

11) The ranking of the candidates using the plurality-with-elimination is . C, B, A, D

D has the fewest first place votes
no 1st place votes to give away

A eliminated next

9 first place votes



$$C: 6 + 26 + 11 = 43$$

$$B: 3 + 5 + 18 = 26$$

eliminate B

end with C

- 12) What is the quota of the weighted voting system $[22: 10, 9, 8, 5, 1]$?
Remember: the notation is $[q: w_1, w_2, w_3, w_4, w_5]$ where w_1 is the weight of player 1 (P1) and w_1, w_2, w_3, w_4, w_5 are written in descending order. The quota is the minimum number of votes required to pass a motion.

22

- 13) Consider the weighted voting system $[32: 28, 3, 2]$.

Dictator: A player that owns enough votes to carry a motion singlehandedly. If the dictator is for the motion, the motion will pass and if the dictator is against the motion, the motion will fail. A player is a dictator if and only if the player's weight is bigger than or equal to the quota.

Is there a dictator? NO

Dummy: A player with votes but zero power.

Is there a dummy? More concretely, is 2 (player 3) the dummy? NO
Do P1 and P2 have all of the power? NO

- 14) In the weighted voting system $[28: 11, 8, 7, 2]$ what is the largest value q can take on?
What are the assumptions? The largest value q can take on is the unanimity of votes.
all of votes

- 15) Please skip. I checked and this is covered in section 2.4.

- 16) In the weighted voting system $[17: 10, 8, 7, 4, 2]$, the total number of possible SEQUENTIAL coalitions is ____.
This question will be moved to the Shapley-Shubik section.

$$\overline{5} \cdot \overline{4} \cdot \overline{3} \cdot \overline{2} \cdot \overline{1}$$

$$5! = 120$$

Refer to the weighted voting system [25: 17, 9, 8, 6] and the Banzhaf definition of power to answer questions #17 - #20.

17) The weight of the coalition {P2, P3, P4}, also written as {9, 8, 6}, is 23. = $9+8+6$

18) The winning coalitions for the weighted voting system are

do you need my votes to pass the motion?

- A) {17, 9, 8, 6}, {17, 9, 8}, {17, 9, 6}, {17, 8, 6} all winners.
- B) {17, 9, 8, 6}, {17, 9, 8}, {17, 9, 6}, {17, 8, 6}, {17, 9} all winners
- C) 17, 9, 8, 6, {17, 9, 8}, {17, 9, 6}, {17, 8, 6}, {17, 9}, {17, 8} all winners
- D) {17, 9, 8, 6}, {17, 9, 8}, {17, 9, 6}, {17, 8, 6}, 9, 8, 6, {17, 9}, {17, 8}, 17, 6 loser
- E) {17, 9, 8, 6}, {17, 9, 8}, {17, 9, 6}, {17, 8, 6}, {9, 8, 6}, {17, 9}, {17, 8}, {17, 6}, {9, 8}, {9, 6}, {8, 6}, {17}, {9}, {8}, {6}

19) Which players in {17, 8, 6} are critical? $\boxed{17} + \boxed{8}$

20) What is the Banzhaf power distribution?

17 : critical 6 times
 9 : critical 2 times
 8 : critical 2 times
 6 : critical 0 times

10 times

$$17 : \frac{6}{10} ; 9 : \frac{2}{10} ; 8 : \frac{2}{10} ; 6 : \frac{0}{10}$$

Refer to the weighted voting system [10: 6, 3, 2] and the Shapley-Shubik definition of power to answer questions #21 - #24.

21) Which player in the sequential coalition $\langle 3, 6, 2 \rangle$ is pivotal?

When looking at a sequential coalition, start counting votes from left to right. At some point there are enough votes to win. The first player that makes this possible is the pivotal player in that coalition. Every sequential coalition has one and only one pivotal player.

2 or P_3

22) How many sequential coalitions are there?

$$3! = 3 \cdot 2 \cdot 1 = 6$$

What are the sequential coalitions?

$\langle 6, 3, 2 \rangle$, $\langle 6, 2, 3 \rangle$, $\langle 3, 6, 2 \rangle$, $\langle 3, 2, 6 \rangle$, $\langle 2, 6, 3 \rangle$, $\langle 2, 3, 6 \rangle$

23) How many times is P1 (denoted by 6) pivotal?

2

24) What is the Shapley-Shubik power distribution?

6 is pivotal 2 times
3 is pivotal 2 times
2 is pivotal 2 times

$$6: \frac{2}{6} ; 3: \frac{2}{6} ; 2: \frac{2}{6}$$