Sample Exam 1A

Use a #2 pencil. Calculators are allowed, but cell phones/ipods etc. are NOT acceptable. Please turn cell phones off.
Choose the alternative that best completes the statement or answers the question and mark your answer on the scantron form. Scantrons with no answers marked will receive a score of 0.

Use the following preference schedule to answer questions 1 to 8.

Number of voters	9	6	4	3	1
First choice	B	A	C	D	C
Second choice	A	D	A	A	A
Third choice	D	C	D	B	B
Fourth choice	C	B	B	C	D

#first place votes
A: 6
B: 9
C: 5
D: 3

1. How many first place votes does candidate C receive?

   A) 6
   B) 5
   C) 9
   D) 3
   E) None of the above.

   4+1=5

2. What is the minimum number of first place votes needed to have a majority?

   A) 23
   B) 11
   C) 12
   D) 9
   E) None of the above.

   9+6+4+3+1=23
   23/2 = 11.5 → round up to ensure a majority.
   (i.e. 12 for, 11 against)

3. Which candidate has a majority of first place votes?

   A) A
   B) B
   C) C
   D) D
   E) None of the above.

   No candidate has 12 or more first place votes

4. Which candidate comes in first using the Plurality Method to rank the candidates?

   A) A
   B) B
   C) C
   D) D
   E) None of the above.

   Plurality method: Who has the most first place votes?

Number of voters	9	6	4	3	1
First choice	B	A	C	D	C
Second choice	A	D	A	A	A
Third choice	D	C	D	B	B
Fourth choice	C	B	B	C	D

Pairwise Comparisons

A vs B: 6 | 9; 4; 3; 1 — 14 vs 9, A wins

A vs C: 9 | 4; 6 | 1; 3 — 18 vs 5

5. How many points does candidate B earn when using the Method of Pairwise Comparisons?

A) 3
B) 2
C) 1
D) 0
E) None of the above.

A vs D: 9 | 3; 6; 4; 1

B vs C: 9 | 6; 3 | 4; 1 — 12 11

B vs D: 9 | 6; 1 | 4; 3 — 10 13

C vs D: 4 | 9; 1 | 6; 3

6. Rank the candidates using the Method of Pairwise Comparisons.

A) A, B, C, D
B) A, C, D, B
C) A, D, B, C
D) B, D, A, C
E) C, B, A, D

A, D, B, C

Points for winning pairs

A: wins all 3, so 3 pts (is also Condorcet Candidate)

B: 1 ; C: 0 ; D: 2

7. Which candidate is the Condorcet Candidate?

A) A
B) B
C) C
D) D
E) None of the above.

wins all head-to-head comparisons

Note: Look at Questions 4 and 7.

8. Which of the following statements is true concerning this preference schedule?

A) The Majority Criterion is violated by the Method of Pairwise Comparisons. (no candidate had a majority.)
B) The Condorcet Criterion is violated by the Method of Pairwise Comparisons. (This never happens)
C) The Condorcet Criterion is violated by the Plurality Method.
D) The Majority Criterion is violated by the Plurality Method. (This never happens)
E) Both the Plurality Method and the Method of Pairwise Comparisons are fair under all 4 criteria.

Use the following preference schedule to answer questions 9 - 12.

Note double check #s 1,2,3,4 are all listed in marked columns

	Number of voters	7	5	3	2
4 points	First choice	A	D	C	B
3 points	Second choice	B	A	D	D
2 points	Third choice	D	B	A	C
1 point	Fourth choice	C	C	B	A

A: 7×4 + 5×3 + 3×2 + 2×1

B: 7×3 + 5×2 + 3×1 + 2×4

C: 7×1 + 5×1 + 3×4 + 2×2

D: 7×2 + 5×4 + 3×3 + 2×3

9. How many points does candidate D earn using the Borda Count method?

A) 51
B) 49
C) 28
D) 42
E) None of the above.

10. Rank the candidates using the Borda Count Method.

A) A, B, C, D
B) D, A, C, B
C) B, D, A, C
D) C, B, D, A
E) None of the above.

A: 51
B: 42
C: 28
D: 49

11. Which candidate is eliminated first using Plurality-With-Elimination?

Number of voters	7	5	3	2
First choice	A	D	C	B

B has fewest 1st place votes.

A) A
B) B
C) C
D) D
E) None of the above.

(note: 17/2 = 8.5 so 9 votes are needed for a majority)

12. Rank the candidates using the Plurality-With-Elimination Method.

A) A, B, C, D
B) D, A, C, B
C) B, D, A, C
D) C, B, D, A
E) None of the above.

Number of voters	7	5	3	2
First choice	A	D	C	~~B~~ D
Second choice	B	A	D	D

With B eliminated, D now has 7 1st place votes. C is the "new loser" = 3rd place.

Number of voters	7	5	3	2
First choice	A	D	D	D
Second choice	~~B~~ D	A	D	D
Third choice	D	~~B~~ D	A	C
Fourth choice	C	C	B	A

With B and C gone, D has 10 1st place votes vs A with 7.

13. Which of the following is NOT a valid quota q for the voting system $[q : 10, 5, 5, 1]$?

A) 11 ok
B) 16 ok
C) 21 ok
D) 26 too big.
E) All of the above.

max quota: 10+5+5+1 = 21

min quota: 21/2 = 10.5, so 11 is minimum

14. In the weighted voting system $[15 : 12, 10, x]$, what is the minimum value for x that guarantees P_3 is NOT a dummy voter?

dummy voter → has no power.

A) $x = 2$
B) $x = 3$
C) $x = 4$
D) P_3 will always be a dummy.
E) None of the above.

Note: 12 and 10 > 15, so look at times when x is solo or paired with 12 or 10.

If x is paired with 12 i.e. $\{12, x\}$ or $\langle 12, x, 10\rangle$ then x needs to be large enough to make $12 + x = 15$. So $x = 3$.

(If you pair x with 10, you'll get a larger # for x, but we are asked for the minimum.)

Use the weighted voting system $[13 : 12, 10, 2]$ and the Banzhaf definition of power to answer questions 15 - 19.

3 players so N=3

15. How many coalitions (not counting the empty coalition) are there?

A) 6
B) 7
C) 8
D) 9
E) None of the above.

$2^N - 1$ so $2^3 - 1 = 8 - 1 = 7$

16. Which players are critical in $\{12, 2\}$?

A) None of the players are critical.
B) All of the players are critical.
C) The player with weight 12 is the only critical player.
D) The player with weight 2 is the only critical player.
E) None of the above.

If player 1 (12 votes) leaves the coalition, is {2} enough? NO → so 12 is critical. Same w/ 2.
Need both to make quota.

17. How many time is the player with weight 10 critical?

A) 0
B) 1
C) 2
D) 3
E) None of the above.

(note 7 coalitions overall!)

{12, 10, 2} {12, 2} {12} {2} losing coalitions
{12, 10} {10, 2} {10}

18. What is the Banzhaf power distribution?

A) $P_1 : \frac{1}{2}; P_2 : \frac{1}{5}; P_3 : \frac{1}{4}$
B) $P_1 : \frac{3}{5}; P_2 : \frac{2}{5}; P_3 : 0$
C) $P_1 : \frac{3}{5}; P_2 : \frac{1}{5}; P_3 : \frac{1}{5}$
D) $P_1 : \frac{3}{7}; P_2 : \frac{2}{7}; P_3 : \frac{2}{7}$
E) None of the above.

P_1: 3/5 P_2: 1/5 P_3: 1/5

number of times a player is critical (underlined)

5 is the total # of times all players are critical (total # of underlines)

19. Modify this weighted voting system by changing the quota. For which quota below will Player 1 have veto power?

A) Player 1 already has veto power.
B) It is impossible for Player 1 to have veto power.
C) Change the quota to 14.
D) Change the quota to 12.
E) Change the quota to 11.

→ P_1 has enough votes to kill a motion (because the remaining votes can't pass a motion: 10 + 2 = 12 < 13.)
AND P_1 does not have 13 or more votes, which would make P_1 a dictator.

Use the weighted voting system $[13 : 12, 10, 2]$ and the Shapley-Shubik definition of power to answer questions 20 - 24.

3 players so $3! = 6$ coalitions

20. Which of the following is a list of all the sequential coalitions?

A) $\langle 12, 10, 2\rangle, \langle 12, 10\rangle, \langle 12, 2\rangle, \langle 10, 2\rangle$
B) $\langle 12, 10, 2\rangle, \langle 12, 10\rangle, \langle 12, 2\rangle, \langle 10, 2\rangle, \langle 12\rangle, \langle 10\rangle, \langle 2\rangle$
C) $\langle 12, 10, 2\rangle, \langle 12, 2, 10\rangle, \langle 2, 10, 12\rangle, \langle 10, 2, 12\rangle$
D) $\langle 12, \underline{10}, 2\rangle, \langle 12, \underline{2}, 10\rangle, \langle 10, \underline{12}, 2\rangle, \langle 10, 2, \underline{12}\rangle, \langle 2, \underline{12}, 10\rangle, \langle 2, 10, \underline{12}\rangle$ has all combinations of 12, 10, and 2
E) None of the above.

21. What player(s) is/are pivotal in $\langle 2, 12, 10\rangle$? Work left to right. Does 2 have enough votes? $2 < 13$ so no.

A) P_1 (12 votes) is the only pivotal player
B) P_2 (10 votes) is the only pivotal player
C) P_3 (2 votes) is the only pivotal player
D) P_1 (12 votes) and P_2 (10 votes) are the only pivotal players
E) None of the above.

Do 2+12 have enough votes? $2+12 = 14 > 13$ so yes.

Note: only 1 pivotal player always!

22. How many times is the player with weight 2 pivotal? (What is the pivotal count?)

A) 0
B) 1
C) 2
D) 3
E) None of the above.

(underlined in # 20)

23. What is the Shapley-Shubik power distribution for the voting system?

A) $P_1 : \frac{1}{2}$; $P_2 : \frac{1}{5}$; $P_3 : \frac{1}{4}$
B) $P_1 : \frac{2}{3}$; $P_2 : \frac{1}{6}$; $P_3 : \frac{1}{6}$
C) $P_1 : \frac{1}{3}$; $P_2 : \frac{1}{3}$; $P_3 : \frac{1}{3}$
D) $P_1 : \frac{5}{6}$; $P_2 : \frac{1}{6}$; $P_3 : 0$
E) None of the above.

P_1: 4/6 P_2: 1/6 P_3: 1/6

times pivotal

times all players pivotal = N!

24. If the number of players increases to 10, how many sequential coalitions would there be?

A) 2^{10}
B) 2^{10-1}
C) $(10-1)!$
D) $10!$
E) None of the above.

$10!$