Exam 1 **MATH 130 Section 001, 8:00 am** **Spring 2015**

Name: Key **Mark Version C**

Use a #2 pencil. Calculators are allowed, but cell phones, i-pods, etc. are NOT acceptable. Please turn cell phones off.
Mark your answer for each question on the scantron form. Scantrons with no answers marked will receive a score of 0.
There are 25 questions but the exam will be scored out of 24 possible correct answers. The maximum possible score is 100%.

Use the following preference schedule to answer questions 1 - 14.

Number of voters	54	42	33	10
First	B	A	C	D
Second	D	C	A	A
Third	C	B	D	C
Fourth	A	D	B	B

1. How many people voted in this election?

A) 13900
B) 4
C) 139
D) 556
E) 54

54
42
33
10
139

2. How many voters selected C as their third choice?

A) 54
B) 64
C) 42
D) 33
E) 10

54
+10
64

3. What is the minimum number of first-place votes needed to have a majority?

A) 70
B) 21
C) 69
D) 68
E) 71

$\frac{139}{2} = 69.5 \uparrow 70$

4. Which candidate has a majority of the first-place votes?

A) A
B) B
C) C
D) D
E) No candidate has a majority of the first-place votes.

no candidate has 70 or more 1ST-place votes

(This is the same preference schedule for use with questions 1 - 14.)

Number of voters	54	42	33	10
First	B	A	C	D
Second	D	C	A	A
Third	C	B	D	C
Fourth	A	D	B	B

5. Rank the candidates using plurality.

   A) C, A, B, D
   B) A, B, C, D
   C) B, A, C, D
   D) B, D, C, A
   E) None of the above.

6. Rank the candidates using the method of pairwise comparisons.

   A) C, A, B, D
   B) A, B, C, D
   C) B, A, C, D
   D) B, D, C, A
   E) C, A, D, B

A vs. B: 42 + 33 + 10 = 85 ✓ (54)

A vs. C: 42 + 10 = 52 (54 + 33 = 87 ✓)

A vs. D: 42 + 33 = 75 ✓ (54 + 10 = 64)

7. How many points does candidate C receive when using the method of pairwise comparisons?

   A) 0
   B) 1
   C) 2
   D) 3
   E) None of the above.

C vs. B: 42 + 33 + 10 = 85 ✓ (54)

C vs. D: 42 + 33 = 75 (54 + 10 = 64) ✓

B vs. D: 54 + 42 = 96 ✓ (33 + 10 = 43)

A: 2 pts.
B: 1 pt.
C: 3 pts
D: 0 pts

8. Which candidate is a Condorcet candidate?

   A) A
   B) B
   C) C
   D) D
   E) There is no Condorcet candidate in this election.

9. Use the results from questions 4 to 8 to identify the true statement below.

   A) Electing the winner using pairwise comparisons violates the majority criterion.
   B) Electing the winner using plurality violates the majority criterion.
   C) Electing the winner using pairwise comparisons violates the Condorcet criterion.
   D) Both plurality and pairwise comparisons are fair elections.
   E) Electing the winner using plurality violates the Condorcet criterion.

no majority candidate - MC cannot be violated

(This is the same preference schedule for use with questions 1 - 14.)

Number of voters	54	42	33	10
First 4	B 216	A 168	C 132	D 40
Second 3	D 162	C 126	A 99	A 30
Third 2	C 108	B 84	D 66	C 20
Fourth 1	A 54	D 42	B 33	B 10

10. Rank the candidates using Borda count.

(A) C, A, B, D
B) A, B, C, D
C) B, A, C, D
D) B, D, C, A
E) C, A, D, B

A: 54+168+99+30 = 351 ✓
B: 216+84+33+10 = 343 ✓
C: 108+126+132+20 = 386 ✓
D: 162+42+66+40 = 310 ✓

11. How many points does B receive when using Borda count?

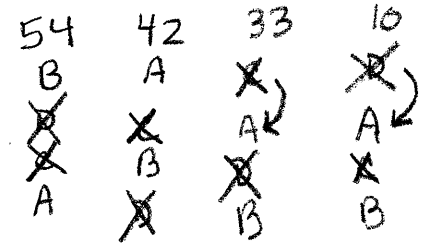
A) 8
B) 147
C) 54
(D) 343
E) None of the above.

12. Which candidate comes in last using plurality-with-elimination?

A) A
B) B
C) C
(D) D has the fewest first-place votes
E) None of the above.

13. When the last place candidate is eliminated, how many first-place votes are transferred and to which candidate?

A) 10 first-place votes are transferred to candidate C.
B) 33 first-place votes are transferred to candidate A.
(C) 10 first-place votes are transferred to candidate A.
D) 33 first-place votes are transferred to candidate D.
E) No first-place votes are transferred.



14. Rank the candidates using plurality-with-elimination.

A) C, A, B, D
(B) A, B, C, D
C) B, A, C, D
D) B, D, C, A
E) C, A, D, B

eliminate D
new count
A: 52 B: 54 C: 33
eliminate C
new count
A: 85 B: 54

15. Which of the following is a valid quota q, assuming no anarchy or gridlock, for the voting system $[q : 12, 6, 3]$? Gridlock:

A) 23 → all players, even if they join forces, could not pass any motions
B) 14
C) 9 → $[8:12,6,3]$ & $[9:12,6,3]$ lead to problems → anarchy
D) 8 → 12 could vote yes, 6 & 3 could vote no
E) All of the above.

16. In the weighted voting system $[15 : x, 11, 2]$, which value for x guarantees that P_1 is a dictator? if the player's weight is bigger than or equal to the quota (15)

A) $x = 5$
B) $x = 14$
C) $x = 16$
D) P_1 can never be a dictator.
E) None of the above.

Use the weighted voting system $[12 : 8, 5, 4]$ and the Banzhaf definition of power to answer questions 17 - 20.

17. What is the weight of the coalition $\{P_2, P_3\}$? $\{5,4\}$

A) 9
B) 17
C) 8
D) 12
E) 13

18. Which of the following is a list all the winning coalitions under the Banzhaf definition of power?

A) $\{8,5,4\},\{8,4,5\},\{5,8,4\},\{5,4,8\},\{4,8,5\},\{4,5,8\}$
B) $\{8,5,4\},\{8,5\},\{8,4\}$
C) $\{8,5\},\{8,4\},\{5,4\}$
D) $\{8\},\{5\},\{4\},\{8,5\},\{8,4\},\{5,4\},\{8,5,4\}$
E) $\{8\},\{5\},\{4\}$

$\{8,5,4\}$ grand coalition ✓
$\{8,5\}$ ✓
$\{8,4\}$ ✓
$\{5,4\}$ loser
$\{8\}$ loser
$\{5\}$ loser
$\{4\}$ loser
$\{\}$ loser

19. Which players are critical in $\{8,5\}$?

A) All of the players are critical.
B) The player with weight 5 is the only critical player.
C) The player with weight 8 is the only critical player.
D) The players with weights 8 and 5 are both critical players.
E) None of the above.

20. What is the Banzhaf power distribution?

A) $P_1 : 2; P_2 : 1; P_3 : 1$
B) $P_1 : \frac{1}{2}; P_2 : \frac{1}{3}; P_3 : \frac{1}{6}$
C) $P_1 : \frac{2}{3}; P_2 : \frac{1}{6}; P_3 : \frac{1}{6}$
D) $P_1 : 3; P_2 : 1; P_3 : 1$
E) $P_1 : \frac{3}{5}; P_2 : \frac{1}{5}; P_3 : \frac{1}{5}$

P_1 (8 votes) : $\frac{3}{5}$
P_2 (5 votes) : $\frac{1}{5}$
P_3 (4 votes) : $\frac{1}{5}$

Use the weighted voting system $[14 : 10, 8, 6]$ and the Shapley-Shubik definition of power to answer questions 21 - 25

21. Which player(s) in the sequential coalition $\langle 8, 6, 10\rangle$ is/are pivotal?

    A) P_1 (10 votes) is the only pivotal player.
    B) P_2 (8 votes) is the only pivotal player.
    C) P_3 (6 votes) is the only pivotal player.
    D) P_1 (10 votes) and P_2 (8 votes) are both pivotal players.
    E) None of the above.

22. Which of the following is a list of all the sequential coalitions?

    A) $\langle 10, \underline{8}, 6\rangle, \langle 10, \underline{6}, 8\rangle, \langle 8, \underline{6}, 10\rangle, \langle 8, \underline{10}, 6\rangle, \langle 6, \underline{8}, 10\rangle, \langle 6, \underline{10}, 8\rangle$
    B) $\langle 10, 8, 6\rangle, \langle 10, 8\rangle, \langle 10, 6\rangle, \langle 8, 6\rangle, \langle 10\rangle, \langle 8\rangle, \langle 6\rangle$
    C) $\langle 10, 8, 6\rangle, \langle 8, 10, 6\rangle, \langle 6, 8, 10\rangle$
    D) $\langle 10, 8, 6\rangle, \langle 10, 8\rangle, \langle 10, 6\rangle, \langle 8, 6\rangle$
    E) None of the above.

23. How many times is P_2 (8 votes) pivotal?

    A) 3
    B) 2
    C) 1
    D) 0
    E) None of the above.

24. What is the Shapley-Shubik power distribution for the voting system?

    A) $P_1 : \frac{4}{6}$; $P_2 : \frac{3}{6}$; $P_3 : \frac{3}{6}$
    B) $P_1 : \frac{2}{6}$; $P_2 : \frac{2}{6}$; $P_3 : \frac{2}{6}$
    C) $P_1 : \frac{4}{6}$; $P_2 : \frac{1}{6}$; $P_3 : \frac{1}{6}$
    D) $P_1 : 1$; $P_2 : 0$; $P_3 : 0$
    E) $P_1 : 2$; $P_2 : 1$; $P_3 : 1$

25. If the number of players increases to 15, how many sequential coalitions will there be?

    A) 15^2
    B) $2^{15} - 1$
    C) $14!$
    D) $15!$
    E) 2^{15}