Reassessing L2

L2: I can evaluate limits numerically, graphically and algebraically.
(Additional practice for Reassessment forms available on the last page(s).)

Basic Preparation

1. Did you do the written practice?
2. Did you do the WeBWorK?
3. Go back to your notes, any handouts from class, the desmos activities, WeBWorK and any written practice you were able to use to prepare. Compare this to your quiz/homework.
4. Do you understand what your mistake was? If so, briefly describe what the mistake is below. If you are unsure, please go to the Calculus Center and work with a tutor until you can describe what your mistake was.

Metacognition

Now, *WHY* did you make the mistake? Answering this question is asking you to think about HOW you think about math (metacognition). Spending time here will help you become more efficient at learning math and is therefore worth the time!

1. Was your incorrect answer due to
   (a) not understanding a concept;
   (b) an error in logical reasoning (e.g., used the correct theorem/test but made the wrong conclusion, used a theorem/test/technique when it did not apply);
   (c) being careless (e.g. not reading directions, not answering the question completely, making arithmetic or basic algebra errors);
   (d) not knowing how to start or formulate an approach to the problem;
   (e) others?

   Briefly describe why your answer was incorrect:

2. What helped you recognize your mistake(s). Here are some examples: the course notes, the textbook, homework or conversations from the Calculus Center. In other words, which strategies for identifying mistakes work well for you and will help you in the future?

3. Rework the ENTIRE PROBLEM. Rewrite your solution from start to finish, carefully fixing the mistake(s) you diagnosed above. By doing the entire problem over again, you can make sure you fix your mistake and better understand the point of the exercise.

4. Describe (in detail) what you have done in order to learn from your mistake(s) and prepare for your next attempt. Did you read the textbook or class notes? Did you look at examples and/or work problems on your own or with your tutor/classmate/instructor, and if so, which problems? Did you take a different approach than listed here? (Again, the point of this isn't just to look at what you did on this problem, but how can you learn from this and be more likely to meet expectations on future assignments on the first try.)

Where topic was first introduced: Module 2. See also Modules 3 and 4.

Evaluating Limits Numerically

Use or create and use a table of values to approximate the limit.

$f(x) = \frac{\sin(x)}{x}$

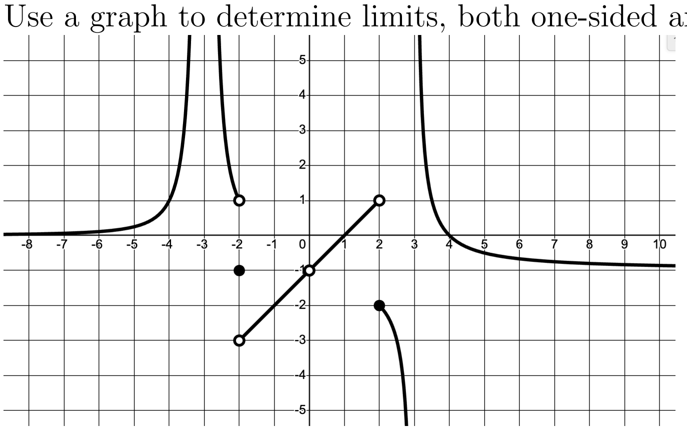
x	$f(x)$
.1	0.99833417
.01	0.99998333
.001	0.99999983
−.001	0.99999983
−.01	0.99998333
−.1	0.99833417

Notice that this table shows values of x approaching 0 from both the left and the right. From the table we approximate $f(x)$ by 0.99, or 0.9999 or even 0.999999 by making x sufficiently close to 0. If we wanted more accuracy, we could make x closer to 0.

This allows us to conclude that $\lim_{x\to 0} \frac{\sin(x)}{x} \approx 1$.

Evaluating Limits Graphically

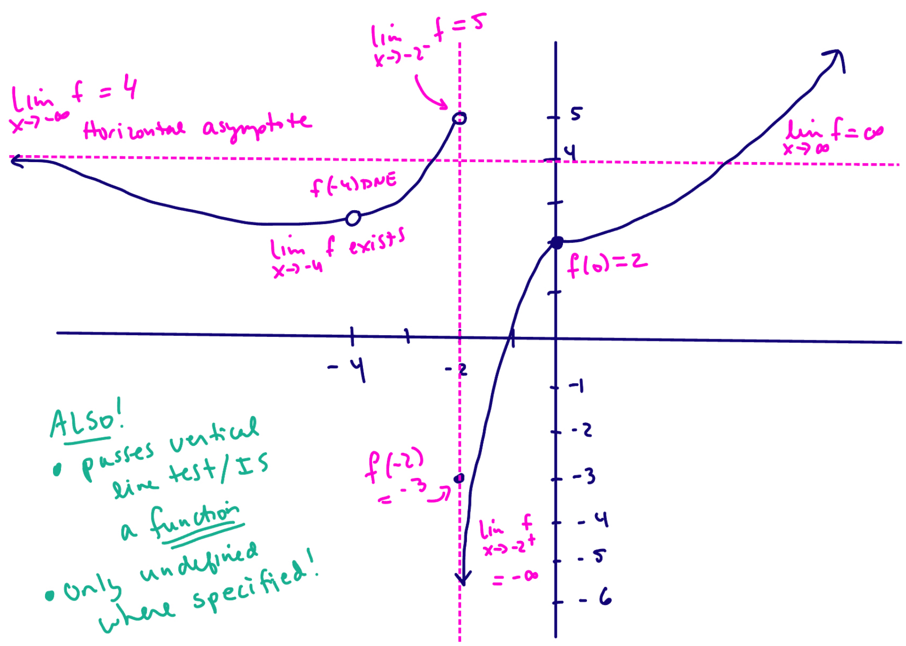
Use a graph to determine limits, both one-sided and two-sided.



1. $\lim_{x\to -\infty} f(x) = 0$
2. $\lim_{x\to -3} f(x) = \infty$
3. $\lim_{x\to -2^-} f(x) = 1$
4. $\lim_{x\to -2^+} f(x) = -3$
5. $\lim_{x\to -2} f(x)$ DNE
6. $\lim_{x\to 0} f(x) = -1$
7. $\lim_{x\to 2^-} f(x) = 1$
8. $\lim_{x\to 2^+} f(x) = -2$
9. $\lim_{x\to 3^-} f(x) = -\infty$
10. $\lim_{x\to 3^+} f(x) = \infty$
11. $\lim_{x\to 3} f(x) =$DNE
12. $\lim_{x\to \infty} f(x) = -1$

Sketch the graph of one function that satisfies the following criteria:

- $\lim_{x\to-\infty} f(x) = 4$
- $f(-4)$ is undefined
- $\lim_{x\to-4} f(x)$ exists
- $\lim_{x\to-2^-} f(x) = 5$
- $f(-2) = -3$
- $\lim_{x\to-2^+} f(x) = -\infty$
- $f(0) = 2$
- $\lim_{x\to\infty} f(x) = \infty$



Evaluating Limits Algebraically

1. Using limit properties:Let $\lim_{x\to3} f(x) = 3$ and $\lim_{x\to3} g(x) = -2$

   (a) $\lim_{x\to3} f(x)g(x) = 3 \cdot -2 = -3$

   (b) $\lim_{x\to3} 4f(x) - 8g(x) = 4 \cdot 3 - 8 \cdot -2 = 28$

   (c) $\lim_{x\to3} \frac{4f(x)}{g(x)} = \frac{4 \cdot 3}{-2} = -6$

   (d) $\lim_{x\to3} (\sin(f(x)) + g(x)) = \sin(\lim_{x\to3} f(x)) + \lim_{x\to3} g(x)) = \sin(3) + -2$

   (e) $\lim_{x\to3} \frac{\sqrt{f(x)+g(x)}}{g(x)} = \frac{\sqrt{\lim_{x\to3} f(x) + \lim_{x\to3} g(x)}}{\lim_{x\to3} g(x)} = \frac{\sqrt{3-2}}{-2} = -1/2$

2. Consider $h(x) = \begin{cases} -x^2+3 & x<2 \\ x+A & x>2 \end{cases}$

What value should A be to make sure that $\lim\limits_{x\to 2} h(x)$ exists? Explain how you know this value of A works and show your work.
$\lim\limits_{x\to 2^-} -x^2+3 = -(2)^2+3 = -1$ and $\lim\limits_{x\to 2^+} x+A = 2+A$. Thus the limit exists when $-1 = 2+A$ or when $A=-3$

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3. Let $k(x) = \begin{cases} \ln(x) & 0<x<2 \\ 5 & x=2 \\ e^{x-2} & x>2 \end{cases}$

Compute the limits:

(a) $\lim\limits_{x\to 0^+} k(x) = \lim\limits_{x\to 0^+} \ln(x) = -\infty$

(b) $\lim\limits_{x\to 1} k(x) = \lim\limits_{x\to 1} \ln(x) = \ln(1) = 0$

(c) $\lim\limits_{x\to 2^-} k(x) = \lim\limits_{x\to 2^-} \ln(x) = \ln(2)$

(d) $\lim\limits_{x\to 2^+} k(x) = \lim\limits_{x\to 2^+} e^{x-2} = e^{2-2} = e^0 = 1$

(e) $\lim\limits_{x\to 2} k(x) =$ DNE because $\ln(2) \neq 1$

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4. Evaluate the limits:

(a) $\lim\limits_{x\to\infty} \frac{1}{x} = 0$

(b) $\lim\limits_{x\to-\infty} \frac{1}{x} = 0$

(c) $\lim\limits_{x\to\infty} \frac{1}{x^2} = 0$

(d) $\lim\limits_{x\to\infty} \left(\frac{1}{x}+4\right) = 4$

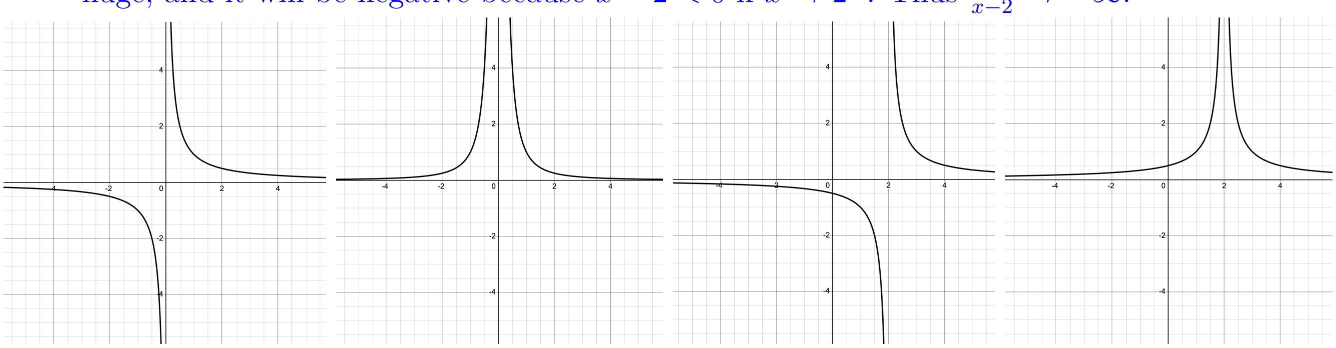
(e) $\lim\limits_{x\to 2^-} \frac{1}{x-2} = -\infty$

(f) $\lim\limits_{x\to 2^+} \frac{1}{x-2} = \infty$

(g) $\lim\limits_{x\to 2} \frac{1}{x-2} =$ DNE

(h) $\lim\limits_{x\to 2} \frac{1}{|x-2|} = \infty$

For these kind of limits, you can reason or use a graph. For example, on part e: As $x\to 2^-$, $x-2\to 0^-$ (negatively). Because the denominator is getting tiny, the overall fraction will get huge, and it will be negative because $x-2<0$ if $x\to 2^-$. Thus $\frac{1}{x-2}\to-\infty$.



Prepare for Reassessment:

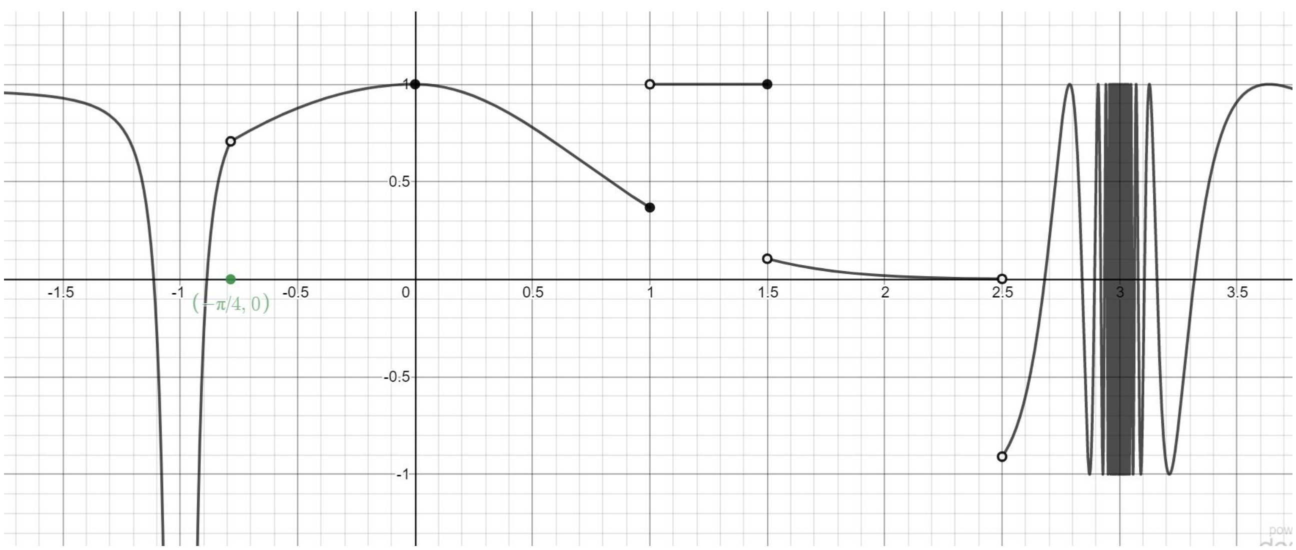
First, reflect on your mistake and the correct solution and what you learned: fill in the blanks "I used to think ________________ but now I think ________________ because I learned ________________."

Second, prepare responses to the following questions:

1. Use the table to estimate $\lim_{x \to 4} f(x)$.

x	$f(x)$
3.9	3.78
3.99	4.17896
3.999	4.18943
4.001	4.18239
4.01	3.8997
4.1	2.709487

2. Estimate the one and two-sided limits when $x = -1.5, -1, -\pi/4, 0, 0.5, 1, 1.5, 2.5, 3$.



3. Sketch the graph of a function such that:

- $\lim_{x \to -\infty} f(x) = \infty$
- $\lim_{x \to -5} f(x) = 10$
- $\lim_{x \to -3} f(x) DNE$
- $\lim_{x \to 0^-} f(x) = -3$
- $\lim_{x \to 0^+} f(x) = \infty$
- $\lim_{x \to 4^-} f(x) = 3$
- $\lim_{x \to 4^+} f(x) = -3$
- $\lim_{x \to \infty} f(x) = 5$

4. Let $\lim_{x \to 4} f(x) = 7$ and $\lim_{x \to 4} g(x) = 5$. Evaluate the following limits.

(a) $\lim_{x \to 4}[f(x) + g(x)] =$

(b) $\lim_{x \to 4}(\ln(f(x)) - g(x)) =$

(c) $\lim_{x \to 4}\left(\frac{f(x)+3}{g(x)}\right) =$

(d) $\lim_{x \to 4}(f(x)g(x)) =$

(e) $\lim_{x \to 4}\left(\frac{\sqrt{g(x)}}{f(x)}\right) =$

5. Let $h(x) = \begin{cases} x^2 + 3x & x < 5 \\ \sqrt{41 - x} & x \geq 5 \end{cases}$. Evaluate $\lim_{x \to 5} h(x)$**algebraically**.

6. Evaluate $\lim_{x \to \infty} 408(0.5)^x$

7. Evaluate $\lim_{x \to \infty} 0.006(5)^x$

8. Evaluate the limits:

(a) $\lim_{x \to \infty} \frac{1}{x^3} =$

(b) $\lim_{x \to -\infty} \frac{1}{x^2} =$

(c) $\lim_{x \to \infty} \frac{1}{x^2} =$

(d) $\lim_{x \to \infty} \left(\frac{1}{x} - 4\right) =$

(e) $\lim_{x \to -3^-} \frac{1}{x+3} =$

(f) $\lim_{x \to -3^+} \frac{1}{x+3} =$

(g) $\lim_{x \to -3} \frac{1}{x+3} =$

(h) $\lim_{x \to -3} \frac{1}{|x+3|} =$