Differential Geometry (Math 8250)

Spring 2013

1 Technicalities

Instructor: Dr. Clayton Shonkwiler (clayton@math.uga.edu)
Office: Boyd 436
Course web page: http://www.math.uga.edu/~clayton/teaching/m8250s13/
Text: *Foundations of Differentiable Manifolds and Lie Groups*, by Frank W. Warner.
Time/Location: 11:15–12:05 MWF, Boyd 326.

2 Summary of the Course

This course develops the theory of differential forms on manifolds and the connections to cohomology by way of de Rham cohomology on the way to stating and proving the Hodge Theorem, which says that every cohomology class on a closed, oriented, smooth Riemannian manifold is represented by a unique harmonic form. The course generally follows Frank Warner's book *Foundations of Differentiable Manifolds and Lie Groups*, though we will skip around a bit and discuss a number of motivations and applications not found in the book.

The Hodge Theorem is a wonderful synthesis of algebraic topology, differential geometry, and analysis which has extensions and applications to algebraic geometry, physics, and data analysis. Proving it will require us to come to terms with concepts ranging from exterior algebras to cochain complexes to the regularity of elliptic operators, so we will get a scenic tour of interesting mathematics along the way.

Moreover, the basic motivation for all of this structure is incredibly simple and down-to-earth. For example, when is a divergence-free vector field (on a closed 3-manifold or even a region in 3-dimensional Euclidean space) the curl of another vector field? The answer turns out to depend on the topology of the manifold or domain; on the 3-sphere or the 3-ball, *every* divergence-free field is a curl, whereas this is *not* true on the 3-torus or on a solid torus. The Hodge Theorem makes this precise and answers analogous questions in all dimensions.

3 Grading

This is an advanced graduate course, so your grade will be based on homework and a final project:

Homework: 70%
Final Project: 20%
Class Participation: 10%

4 Attendance

The University requires that students attend class, and warns that students who miss more than 6 classes may be withdrawn from the course by the instructor.

5 Disclaimer

The course syllabus is a general plan for the course; deviations announced in class may be necessary.

6 Anticipated Schedule

Topic	**Chapter**	**Weeks**
Manifolds	1	2
Differential Forms	2	4
Integration on Manifolds	4	2
de Rham Cohomology	5	3
The Hodge Theorem	6	4