Introduction to Differential Manifolds (Math 670)

Spring 2025

Technicalities

Instructor: Clayton Shonkwiler (clayton.shonkwiler@colostate.edu)
Office: Weber 206C
Course web page: https://www.math.colostate.edu/~clayton/teaching/m670s25/
Course notes: https://shonkwiler.org/dg
Time/Location: 12:00–12:50 Monday, Wednesday, and Friday in Scott Bioengineering 231.
Office Hours: By appointment.

Summary of the Course

The course will be an introduction to differentiable manifolds with an eye towards Lie groups and homogeneous spaces, as well as toward Riemannian geometry.

We will start with the basics of differentiable manifolds (tangent spaces, vector fields, Lie brackets, etc.) and come to grips with differential forms and tensors, including Riemannian metrics. With that background under our belts we will be able to dive into the study of Lie groups and homogeneous spaces. Lie groups are groups which are also manifolds – such as matrix groups like $GL(n, \mathbb{R})$ and $SU(n)$ – and they are central to much of modern mathematics. Homogeneous spaces are manifolds which arise as quotients of Lie groups – for example, Stiefel manifolds and Grassmannians – and many manifolds of interest in practice are homogeneous manifolds.

Finally, we will dive deeper on Riemannian metrics, which give notions of distance and curvature on manifolds, and see some ways they arise in applications.

Some familiarity with point-set topology, multivariable calculus, and linear algebra will be very helpful, but otherwise the course should be reasonably self-contained.

There is no official text for the course, but the following books may be useful resources:

- *Foundations of Differentiable Manifolds and Lie Groups*, by Frank W. Warner
- *Calculus on Manifolds* and *A Comprehensive Introduction to Differential Geometry*, by Michael Spivak
- *Differential Geometry and Lie Groups I & II*, by Jean Gallier and Jocelyn Quaintance
- *Differential Topology*, by Victor Guillemin and Alan Pollack
- *Lectures on Symplectic Geometry*, by Ana Cannas da Silva
- *Riemannian Geometry*, by Manfredo Perdigão do Carmo

1 Grading

This is an advanced graduate course, so your grade will be based on homework and a final project:

Homework: 60%
Final Project: 30%
Class Participation: 10%

2 Disclaimer

The course syllabus is a general plan for the course; deviations announced in class may be necessary.

3 Anticipated Schedule

Topic	**Weeks**
Manifolds and Vector Fields	2
Differential Forms and Tensors	3
Lie Groups and Homogeneous Spaces	5
Riemannian Metrics and Applications	4