Introduction to Differential Manifolds (Math 670)

Spring 2021

Technicalities

Instructor: Clayton Shonkwiler (clay@shonkwiler.org)
Office: Virtual
Course web page: https://www.math.colostate.edu/~clayton/teaching/m670s21/
Time/Location: 3:00–3:50 Monday, Wednesday, Friday, online.
Office Hours: Tuesday 3:00–4:00 and Wednesday 4:00–5:00.

Summary of the Course

The course will be an introduction to differentiable manifolds with an eye towards Lie groups and homogeneous spaces, as well as toward symplectic geometry.

We will start with the basics of differentiable manifolds (tangent spaces, vector fields, Lie brackets, etc.) and come to grips with differential forms and tensors. With that background under our belts we will be able to dive into the study of Lie groups. Lie groups are groups which are also manifolds – such as matrix groups like $GL(n, \mathbb{R})$ and $SU(n)$ – and they are central to much of modern mathematics.

Building on this knowledge of Lie groups and their associated Lie algebras, we will finish by introducing symplectic geometry in the setting of coadjoint orbits and their natural Kirillov–Kostant–Souriau symplectic form. While it arises quite naturally in this setting, symplectic geometry has its roots in Hamiltonian mechanics and is relevant to a very large number of pure and applied areas of mathematics.

Some familiarity with point-set topology, multivariable calculus, and linear algebra will be very helpful, but otherwise the course should be reasonably self-contained.

There is no official text for the course, but the following books may be useful resources:

- *Foundations of Differentiable Manifolds and Lie Groups*, by Frank W. Warner
- *Calculus on Manifolds* and *A Comprehensive Introduction to Differential Geometry*, by Michael Spivak
- *Differential Geometry and Lie Groups I & II*, by Jean Gallier and Jocelyn Quaintance
- *Differential Topology*, by Victor Guillemin and Alan Pollack
- *Lectures on Symplectic Geometry*, by Ana Cannas da Silva
- *Introduction to Symplectic Topology*, by Dusa McDuff and Dietmar Salamon

1 Grading

This is an advanced graduate course, so your grade will be based on homework and a final project:

Homework: 60%
Final Project: 30%
Class Participation: 10%

Homework will be collected regularly throughout the semester. For the final project, you will investigate a connection between material from this class and some other area of mathematics

(combinatorics, probability, applied math, etc.) and write a short paper and give a 15-minute talk about this connection.

2 Disclaimer

The course syllabus is a general plan for the course; deviations announced in class may be necessary.

3 Anticipated Schedule

Topic	**Weeks**
Manifolds and Vector Fields	2
Differential Forms and Tensors	3
Lie Groups	5
Symplectic Geometry	4