Groups as Manifolds: An Introduction to Matrix Groups (Math 476)

Spring 2021

Technicalities

Instructor: Clayton Shonkwiler (clay@shonkwiler.org)
Office: Virtual
Course web page: https://www.math.colostate.edu/~clayton/teaching/m476s21/
Text: *Matrix Groups for Undergraduates*, by Kristopher Tapp.
Time/Location: 12:30–1:45 Tuesdays and Thursdays, online.
Office Hours: Tuesday 3:00–4:00 and Wednesday 4:00–5:00.

Course Content

Part I: Matrices and matrix groups – 4 weeks

- (Tapp Chapter 1) Rigid motions, fields and skew-fields, quaternions, general linear groups, conjugation.
- (Tapp Chapter 2) Real matrix groups, complex and quaternionic matrices as real matrices.
- (Tapp Chapter 3) Inner products, orthogonal groups, isometries, symmetry groups.
- (Tapp Chapter 4) Topology, continuity, connectedness, compactness.

Part II: Algebraic and geometric structures – 5 weeks

- (Tapp Chapter 5) Lie algebras, vector fields, o_n.
- (Tapp Chapter 6) Series, exponentiation, integral curves, one-parameter subgroups.
- (Tapp Chapter 7) Matrix groups as manifolds, general manifolds, examples.
- (Tapp Chapter 8) Lie brackets, adjoint representation, double covers.

Part III: Structure and Symmetry – 6 weeks

- (Tapp Chapter 9) Maximal tori, shapes, rank of a compact matrix group.
- (Tapp Chapter 10) Projective spaces, group actions, homogeneous manifolds, Lie groups.
- (Tapp Chapter 11) Roots, structure, Weyl group, classification.

Prerequisites

MATH 261, MATH 366, MATH 369.

Course Goals

Matrix groups are crucially important in areas ranging from computer graphics to quantum information theory to Riemannian geometry. As groups they are algebraic objects, usually realized as the group of symmetries of some space, but matrix groups are also geometric objects in their own right: they are all manifolds, which are higher-dimensional analogs of smooth surfaces. The goal in this course is to provide a geometric introduction to matrix groups, including orthogonal and unitary groups, Lie algebras, and the exponential map.

Assignments

It is impossible to *learn* mathematics without actually *doing* mathematics. The goal of the assignments is to deepen your understanding of the concepts, tools and techniques discussed in class, as well as to give you the opportunity to practice explaining your mathematical thinking. The importance of effective communication is vital: knowledge without the ability to communicate that knowledge is of limited value. As such, to get full credit on a problem your solution must be clear and well-written.

Reading Assignments

You will be assigned to read relevant sections from the text before the material is discussed in class. Although it is impossible (at least absent some pretty intrusive surveillance) to check whether you are doing this reading, you will also be expected to do a short online reading comprehension quiz on Canvas for each reading assignment. This quiz will not be graded for correctness, only for whether you made an honest attempt at each problem.

Mini-Presentations

Most Thursday classes will be devoted to mini-presentations, where you will give examples, prove lemmas, and present visualizations. These are not intended to be particularly polished or comprehensive: they will reflect the messy realities of learning and using challenging mathematics. These mini-presentations will (mostly) be assigned randomly.

Homework

Homework will be collected every week or so.

Homework must be stapled with your name clearly written at the top. What you turn in should be a final copy: it should be neat, legible, and well-organized. If I can't read or understand your work you won't receive any credit.

Late homework will not be accepted, so you should turn in whatever you have completed on the due date in order to get credit for it.

I strongly encourage you to work on solving homework problems with your fellow classmates. However, the work you turn in must reflect your own knowledge and understanding and not that of anyone else. Therefore, you *must* write up your solutions yourself.

Final Project

There will be a final project in this course, the format of which is up to you. One option is to create a poster for the math department poster session (assuming it happens, it will probably be held on **Thursday, May 6**). Other options include writing a paper, or creating a (well-documented) program, or creating a video, but this is not intended to be an exclusive list: please talk to me if you have an idea and want to see if it can be made into a viable final project. The following are deadlines for various milestones along the way:

Feb. 26: Project topic
Mar. 19: Two-page project description

Apr. 9: Rough draft
May 7: Final draft

You may work in a group of no more than 3 people.

Attendance

You are expected to attend class every day, to participate in class, to read the textbook, and to do the homework.

Grading

Your final grade in the course will be determined by:

Reading Assignments: 5%
Mini-Presentations: 15%
Homework: 50%
Final Project: 30%

Here's how the grading process works. First, I compute an overall course grade for you on a scale of 0–100 by combining your homework, presentation, and project grades using the weights above. Then, I rank everybody in the class in order by their score and assign cutoffs for 'A', 'B', 'C', and 'D'. Generally these are somewhat lower than the traditional 90, 80, 70, and 60. When setting the cutoff I consider the students immediately above and below the line and try to take into account improvement and other circumstances. That being said, the list is never, ever reordered. Regardless of other circumstances, a better score in the class should always earn at least as good a letter grade. Ultimately, I can only grade the course based on what you have actually done.

Policies

The Department of Mathematics has a set of policies which cover topics ranging from cell phones to alternate exams. These are available at
https://mathematics.colostate.edu/undergraduate-students/departmental-class-policies/
and it is your responsibility to read them.

Some particular issues of interest:

Academic honesty

Colorado State University has an Academic Integrity Policy and Student Conduct Code; you can read about this policy at http://policylibrary.colostate.edu/policy.aspx?id=442, and find related materials at https://tilt.colostate.edu/Undergrad. This will be enforced in Math 476. Briefly, while you are encouraged to seek out help, including from your peers, for homework assignments, work that you represent as your own must, in fact, be your own. Cases of flagrant academic dishonesty will be brought to the attention of the TILT Academic Integrity Program.

SDC

Colorado State University is committed to providing reasonable accommodations for all individuals with disabilities; the Student Disability Center (https://disabilitycenter.colostate.edu) coordinates the necessary support systems.

If you need accommodation it is up to you to work with SDC to make suitable arrangements; the sooner you do this, the better.

Additional help

If I were a perfect teacher, you could learn everything you need to know just by going to class and doing the assignments. Unfortunately, I am not a perfect teacher, so there's a good chance that, at some point, you'll find yourself confused, stuck or otherwise frustrated by the material or the course. If you do, ask for help! Come to office hours, send me email, ask me questions after class.

Also, your fellow classmates are a great resource! Odds are that, for any question you have, there's someone in the class who can answer it, so don't be afraid to ask. Even the simple process of explaining why you're stuck to someone who is just as confused as you is often enough to make things clearer. Just be sure to return the favor when you get the chance to help someone else.

If you need additional help or outside resources, please ask and I will be happy to try to give suggestions.

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Disclaimer

The course syllabus is a general plan for the course; deviations announced in class may be necessary.