Introduction to Differential Geometry (Math 474)

Fall 2021

Technicalities

Instructor: Dr. Clayton Shonkwiler (clayton.shonkwiler@colostate.edu)
Office: Weber 206C
Course web page: http://math474.com
Text: *Differential Geometry: A First Course in Curves and Surfaces*, by Theodore Shifrin, which is available free online at http://math474.com/shifrin
Time/Location: 2:00–2:50 MWF, Engineering B4
Office Hours: Wednesday 1:00–2:00 and Thursday 9:30–10:30 in Weber 017

Course Content

Part I: Curve Theory – 6 weeks

- (Shifrin 1.1–1.2) Local theory of curves in the plane and in 3-space: Parametrized curves, arc length, curvature and torsion, Frenet frame.
- (Shifrin 1.2, notes) Advanced topics in local theory: Local canonical form, higher dimensions, Bishop frame, approximations by curves of constant curvature and torsion.
- (Shifrin 1.3, notes) Global differential geometry of curves in $\mathbb{R}^3$: Fundamental theorem, Crofton's formula, Fenchel's theorem, the Fáry–Milnor theorem, Link = Twist + Writhe.
- (Shifrin 1.3, notes) Global differential geometry of plane curves: Hopf Umlaufsatz, Four Vertex Theorem, converse to the Four Vertex Theorem, Fabricius–Bjerre theorem, isoperimetric inequality.
- **Midterm 1**: Oct. 1–8.

Part II: Local Theory of Surfaces – 7 weeks

- (Shifrin 2.1) Regular surfaces and parametrizations: Definitions, examples, differentiable curves on surfaces, differentiable functions on surfaces.
- (Shifrin 2.1) Tangent planes and first fundamental form: Differential, tangent plane, intrinsic definitions, inner products, surface area.
- (Shifrin 2.2) Gauss map: Definition, second fundamental form, curvature of curves on surfaces, principal curvatures, Gaussian and mean curvatures.
- (Shifrin 2.3) Theorema Egregium: Christoffel symbols, Codazzi equations, Gauss equations, proof that Gaussian curvature is an isometry invariant, fundamental theorem.
- (Shifrin 2.4) Parallel transport and geodesics: Vector fields, covariant derivative, parallel fields, geodesics and distance, Clairaut's relation.
- **Midterm 2**: Nov. 19–Dec. 3.

Part III: Global Theory of Surfaces – 2 weeks

- (Shifrin 3.1, notes) The Gauss–Bonnet theorem: Triangulations, local Gauss–Bonnet, Euler characteristic, global Gauss–Bonnet, consequences.
- (Shifrin 3.2, notes) The hyperbolic plane: Models of the hyperbolic plane, geodesics, fractional linear transformations, Einstein's rotating disk.

Prerequisites

Students are expected to have a solid grounding in multivariable calculus and linear algebra, equivalent to the content offered in MATH 261 and MATH 369. Students without experience in proof-based courses (e.g., Math 317 or Math 366) may struggle.

Course Goals

Students will develop a deep understanding of the differential geometry of curves and surfaces, including the various relevant notions of curvature. This will require a synthesis of geometric visualization, symbolic and numerical calculation, and rigorous reasoning and communication. At the end of the course, students should be prepared for a graduate course in the differential geometry of manifolds.

Additional Resources

In addition to the official text and the course notes, these books may also be useful references:

- *Differential Geometry of Curves and Surfaces*, by Manfredo P. Do Carmo
- *Elementary Differential Geometry*, by Barrett O'Neill
- *Elementary Differential Geometry*, by Andrew Pressley
- *Differential Geometry of Curves and Surfaces*, by Thomas Banchoff and Stephen Lovett
- *Geometry from a Differentiable Viewpoint*, by John McCleary

Assignments

It is impossible to *learn* mathematics without actually *doing* mathematics. The goal of the assignments is to deepen your understanding of the concepts, tools and techniques discussed in class, as well as to give you the opportunity to practice explaining your mathematical thinking. The importance of effective communication is vital: knowledge without the ability to communicate that knowledge is of limited value. As such, to get full credit on a problem your solution must be clear and well-written.

Reading Assignments

You will be assigned to read relevant sections from the text before the material is discussed in class. Although it is impossible (at least absent some pretty intrusive surveillance) to check whether you are doing this reading, you will also be expected to do a short online reading comprehension quiz on Canvas for each reading assignment. This quiz will not be graded for correctness, only for whether you made an honest attempt at each problem.

Homework

Homework will be collected every week or two.

Homework must be stapled with your name clearly written at the top. What you turn in should be a final copy: it should be neat, legible, and well-organized. If I can't read or understand your work you won't receive any credit.

Late homework will not be accepted, so you should turn in whatever you have completed on the due date in order to get credit for it.

I strongly encourage you to work on solving homework problems with your fellow classmates. However, the work you turn in must reflect your own knowledge and understanding and not that of anyone else. Therefore, you *must* write up your solutions by yourself.

Exams

There will two take-home midterm exams. You will have approximately 1 week to do each of the take-home midterms; the first will be due on **October 8** and the second on **December 3**.

No make-up examinations will be given in the course.

Final Project

You will create a final project in a format of your choosing; one option is to create a poster for the math department poster session. The poster session will (probably) be held on Thursday, December 9. A list of potential project ideas will be circulated; you are also very welcome to develop your own. The following are deadlines for various milestones along the way:

Oct. 1: Project proposal
Oct. 29: Two-page project description
Nov. 19: Project rough draft
Dec. 9: Project final draft

You may work on your poster in groups of up to three people.

Attendance

You are expected to attend class every day, to participate in class, to read the textbook, and to do the homework.

Grading

Your final grade in the course will be determined by:

Homework and Class Participation: 40%
Exams: 40%
Final Project: 20%

Here's how the grading process works. First, I compute an overall course grade for you on a scale of 0–100 by combining your exam and homework grades using the weights above. Then, I rank everybody in the class in order by their score and assign cutoffs for 'A', 'B', 'C', and 'D'. Generally these are somewhat lower than the traditional 90, 80, 70, and 60. When setting the cutoff I consider the students immediately above and below the line and try to take into account improvement and other circumstances. That being said, the list is never, ever reordered. Regardless of other circumstances, a better score in the class should always earn at least as good a letter grade. Ultimately, I can only grade the course based on what you have actually done.

Policies

The Department of Mathematics has a set of policies which cover topics ranging from cell phones to alternate exams. These are available at

https://mathematics.colostate.edu/undergraduate-students/departmental-class-policies

and it is your responsibility to read them.

Some particular issues of interest:

Academic honesty

Colorado State University has an Academic Integrity Policy (https://catalog.colostate.edu/general-catalog/policies/students-responsibilities) and a Student Conduct Code (https://resolutioncenter.colostate.edu/student-conduct-code/). These will be enforced in Math 474. Briefly, while you are encouraged to seek out help, including from your peers, for homework assignments, all work on any exam must be your own. Cases of flagrant academic dishonesty will be referred to Student Conduct Services.

Student Disability Center

Colorado State University is committed to providing reasonable accommodations for all individuals with disabilities; the Student Disability Center (https://disabilitycenter.colostate.edu) coordinates the necessary support systems.

If you need accommodation, especially for exams, it is up to you to work with the Student Disability Center to make suitable arrangements; the sooner you do this, the better.

Additional help

If I were a perfect teacher, you could learn everything you need to know just by going to class and doing the assignments. Unfortunately, I am not a perfect teacher, so there's a good chance that, at some point, you'll find yourself confused, stuck or otherwise frustrated by the material or the course. If you do, ask for help! Come to office hours, send me email, ask me questions after class.

Also, your fellow classmates are a great resource! Odds are that, for any question you have, there's someone in the class who can answer it, so don't be afraid to ask. Even the simple process of explaining why you're stuck to someone who is just as confused as you is often enough to make things clearer. Just be sure to return the favor when you get the chance to help someone else.

If you need additional help or outside resources, please ask and I will be happy to try to give suggestions.

Copyright

Disclaimer

The course syllabus is a general plan for the course; deviations announced in class may be necessary.